MODEL INDEPENDENT ECONOMICS BASED ON SWARM ENGINEERING

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ABSTRACT. In the classical approach to economics, conclusions about the way in which economic systems behave are drawn from the analysis of the consequences of assumptions about how the system constituents (individuals, firms, governments, etc.) will behave. Like other nonlinear dynamic systems, the analysis of such systems is difficult, and the detailed behaviors of the systems are highly dependent on the systems' initial conditions and specific design. Small changes in parameters can have very significant effects on the detailed function of the system. Careful scrutiny of the assumptions and initial conditions must be undertaken to ensure that the conclusions are relevant to the real world. Such an approach represents an if-then approach to economics, and has a specific weakness in that it is highly dependent on the creativity of the group generating the assumptions.

We describe an alternative, only-if, approach to economics. Adapted from swarm engineering, this approach is a model-independent approach to economics. We demonstrate how, using a particularly simple model, the global economic goal can be described mathematically and used to determine a condition under which the global goal can be reached. We then provide several different types of vendor/consumer agent pairs which satisfy this goal. We also provide an example of a simple system and global goal that is impossible to achieve, demonstrating how this can be determined in a model-independent way.

1. INTRODUCTION

Economic systems are systems made up of a large number of autonomous agents interacting in pairs and groups. The interactions between agents is generally complex, nonlinear, and difficult to analyze. In order to understand what global outcomes these interactions will generate, a great deal of effort must be expended using complex mathematical tools. The main difficulty of such an analysis is that they are valid only within the scope of the assumptions governing the interactions and the behaviors of the agents. Deviations from these behaviors or assumptions requires that the system be analyzed again, and the results can vary widely from those of even closely related systems. Much of the disagreement between economists derives from the detailed assumptions about relatively minute aspects of the economic system or the interactions between agents.

Recently, much work has been done on generating understanding of economic systems using computational models of economics[5, 6, 7, 4]. The advantage of using such systems is that global trends can be generated from complex local interactions of agents that are independent, thereby freeing the user from some of the complexity of solving complex mathematical equations that are difficult to solve, and may or

Key words and phrases. swarm engineering, swarm economics.

may not pertain to the varied economic situations one might wish to examine. By making subtle changes to the behaviors of agents, researchers can explore how global changes are affected by local interactions, communication methodologies, and various technologies in use at different points in economic systems.

The main drawback with these systems is that the conclusions are limited to the scope of the systems that have been either envisaged by researchers theoretically or implemented in computational models. However, economic systems are, by their nature, nonlinear dynamic systems, subject to wide variations in behavior due to things like potentially small changes in the behaviors of individuals or information, however erroneous, that moves through the system. It is advantageous, then, to think about the creation of economic theory based, not on models of human behavior, but on economic principles that transcend human behavior. If such a theory might exist, conclusions about how to get to specific desired outcomes might be made in the place of analysis of how behaviors result in specific, potentially undesirable, outcomes.

This chapter examines a method based on swarm engineering of generating global requirements for economic systems that are not based on the model of the agents. We examine, in particular, how the model yields the requirements for the behaviors of agents in order to achieve specific global outcomes. This is accomplished in the case of a very simple economic system, though our results are intended to be general enough to apply to other economic systems. We examine the limits of the system, determining methodologies for examining whether or not a particular outcome is possible.

2. Model Free Analysis of a Simple Economic System

We start with an analysis of the equations that govern simple economic systems. For extremely simple economic systems, consisting of a single commodity, and a set of consumers and vendors of the commodity, we assume that there are several quantities that consumers can measure and several quantities that vendors can measure.

2.1. Basic definitions.

Vendors concentrate primarily on purchasing or producing commodities, and presenting them for sale. Vendors are aware of the cost of purchasing or producing commodities (c_c), the fixed costs associated with maintaining a store front (virtual or otherwise)(c_f), the price they are charging (p), the number of consumers regularly entering their store (N_c), and the likelihood that they'll make a sale at a given price (after making a few sales)($g(p)$). If the vendor does not utilize other communication or information gathering techniques or tools, this is the complete set of things that can be directly measured.

Consumers concentrate primarily on purchasing and subsequently consuming commodities, spending resources that may be subject to a regular, irregular, or nonexistent replenishment activity. Each consumer is governed by a likelihood to purchase the commodity ($g(p)$) which is a function of many different things including the nature of the consumer (internal state which may include mood, perceptions of various things in the world, the amount of money one has, perception of necessity of the commodity, etc.). Consumers can measure the price of the commodity (p), the inclusive cost of the commodity (for instance, by observing vendors) ($c_v = c_c + c_f$), the resources in their possession (m). In physical locations, consumers can get an idea of how many other consumers are purchasing the same products (N_{c,v_i}), which is a vague measure of the global demand (D).

Other indirect quantities can be measured by vendors and/or consumers. For instance,

$$I = \left(\alpha\frac{\frac{dC}{dt}}{C} + \beta\frac{\frac{dP}{dt}}{P}\right) = \left(\frac{\alpha}{2}\left(\frac{\frac{dc_f}{dt}}{c_f} + \frac{\frac{dc_c}{dt}}{c_c}\right) + \frac{\beta}{N_v}\sum_{i=1}^{N_v}\frac{\frac{dp_i}{dt}}{p_i}\right) \tag{1}$$

may be identified as the inflation rate. α and β are the relative importance of the two parts of the economy. $\alpha+\beta=1$. Note that if $\frac{\frac{dC}{dt}}{C} = \frac{\frac{dP}{dt}}{P}$ then the prices and fixed costs are increasing at the same rate, so that relative profits remain unchanged.

Global demand for a product at a price p can be calculated by

$$D(p) = \sum_{i=1}^{k}\sum_{j=1}^{N_{c,i}} g_j(p) \tag{2}$$

where $N_{c,i}$ is the number of customers per time period that enter the i^{th} establishment and $g_j(p)$ is the probability that customer j will purchase a product at the price p. If prices are not necessarily constant across vendors, the demand may be denoted by

$$D = \sum_{i=1}^{k}\sum_{j=1}^{N_{c,i}} g_j(p_i)\,. \tag{3}$$

Interestingly, demand is an emergent property [1], as $\frac{\partial g}{\partial D} = 0$; the individual consumer's likelihood to purchase a product at a given price is in no way influenced by the overall global demand for the product. As a result, this and many other properties may be identified as emergent properties of the group of consumers. As a result, swarm engineering is a reasonable approach to this kind of problem in the design of economic systems.

Economic systems are defined by their constituent parts and the differential relations between them. We can examine the differential relations between them generally in order to explore the way in which economic systems behave. $\frac{\partial c_c}{\partial c_f}$ denotes the relation between the production and fixed costs of production of a commodity. The detailed value of this relation cannot be examined, though some conclusions might be in order, as detailed below.

The time derivatives of the various components give information about the various quantities. In Table 1, we list the various quantities identified above, and determine their identities and some of their relationships.

Quantity	**Time Derivative**	**Identification**
c_c	$\frac{dc_c}{dt}$	component of Inflation
c_f	$\frac{dc_f}{dt}$	component of Inflation
$c_v = c_c + c_f$	$\frac{dc_v}{dt}$	component of Inflation
p	$\frac{dp}{dt}$	component of Inflation
N_c	$\frac{dN_c}{dt}$	component of demand
g_p	$\frac{dg_p}{dt}$	component of demand
m	$\frac{dm}{dt}$	$in - E$
D	$\frac{dD}{dt}$	change in demand
in	$\frac{din}{dt}$	rate of income growth
E	$\frac{dE}{dt}$	individual expenditure change
$pr = p - c_v$	$\frac{dpr}{dt} = \frac{dp}{dt} - \frac{dc_v}{dt}$	rate of change of profit
I	$\frac{dI}{dt}$	change in inflation
N_{c,v_i}	$\frac{dN_{c,v_i}}{dt}$	component of demand

Table 1: This table lists the various quantities that vendors and consumers can measure, and describes their time derivatives.

The quantities identified in Table 1, along with their identities are important for later work, as we begin to understand what can and cannot be said about economic systems without including specific models.

We look next at the relationship between the various quantities. Table 2 gives the various partial derivatives of the quantities given in Table 1. Of interest to us, and what defines these economic systems, is the relationship of each variable to the other variables in the system. As an example, consider the differential relation $\frac{\partial c_v}{\partial c_f}$. This quantity is generally a positive quantity in economic systems because an increase in the fixed cost creates an increase in the overall vendor cost. In particular,

$$\frac{\partial c_v}{\partial c_f} = 1. \tag{4}$$

On the other hand, suppose that we consider $\frac{\partial D}{\partial c_f}$. This value is not easy to determine. Some consumers might have no response or a positive response to the vendor costs decreasing. On the other hand, the consumers might have a negative response if they are aware of the change, as the vendors might not then pass it on to the consumers. It all depends on the consumer, which is not something that is uniform. Therefore, we conclude the

$$\frac{\partial D}{\partial c_f} =?. \tag{5}$$

We list these relations in Table 2

/	c_c	c_f	c_v	p	N_c	N_{c,v_i}	g_p	m	in	E	D	pr	I
c_c	1	0	1	≥ 0	?	?	?	0	0	0	?	-1	≥ 0
c_f	0	1	-1	≥ 0	?	?	?	0	0	0	?	-1	≥ 0
c_v	1	-1	1	≥ 0	?	?	?	0	0	0	?	-1	≥ 0
p	0	0	0	1	?	?	?	0	0	?	?	1	≥ 0
N_c	≤ 0	≤ 0	≤ 0	?	1	?	?	0	0	?	?	0	0
N_{c,v_i}	≤ 0	≤ 0	≤ 0	?	≥ 0	1	?	0	0	?	?	0	0
g_p	0	0	0	?	?	?	1	0	0	?	≥ 0	0	0
m	0	0	0	0	?	?	?	1	0	?	?	0	0
in	0	0	0	0	?	?	?	1	1	?	?	0	0
E	0	0	0	0	?	?	?	-1	0	1	?	0	0
D	0	0	0	?	?	?	≥ 0	0	0	?	1	0	?
pr	0	0	0	0	?	?	?	0	0	0	?	1	0
I	0	0	0	?	?	?	?	0	0	?	?	0	1

Table 2: This gives the differential relations between the various quantities.

Note in this table that there are specific numbers such as 1 or -1. In these cases, there are direct functional relations between the various elements of the system, and the partial derivative can be directly calculated. On the other hand, often times there are ranges such as ≥ 0. In this case, the value is non-negative, but it's exact value cannot be generally pinned down. If there is a 0 value, this indicates no direct relation between the variables. Finally, a ? value indicates that the value or range cannot be determined because this is a function of the behaviors of the agents which cannot be generally predicted. These indicate the areas where the system designer has the opportunity to exert control.

We may now proceed to examine the system using these differential relations.

2.2. **Model independent stationary price.** One of the measurables we might interest ourselves in is the average price of a commodity. In our one-commodity system, the average price is directly related to things such as inflation, profit margin, and value. The average price of a commodity may be calculated as follows.

$$P = \frac{1}{N_v} \sum_{i=1}^{N_v} p_i. \tag{6}$$

where N_v is the number of vendors in the system and each p_i is vendor i's price. In this case, the rate of change of the price may be calculated as

$$\frac{dP}{dt} = \frac{1}{N_v} \sum_{i=1}^{N_v} \left(\sum_{j=1}^{N_p} \frac{\partial p_i}{\partial K_j} \frac{dK_j}{dt} \right), \tag{7}$$

where N_P is the number of other properties and each K_j is a property. Referring back to Table 2 and ignoring derivative or emergent properties, we can expand this equation to

$$\frac{dP}{dt} = \frac{1}{N_v} \sum_{i=1}^{N_v} \left(\frac{\partial p_i}{\partial c_v} \frac{dc_v}{dt} + \frac{\partial p_i}{\partial N_{c,v_i}} \frac{dN_{c,v_i}}{dt} + \frac{\partial p_i}{\partial g_p} \frac{dg_p}{dt} + \frac{\partial p_i}{\partial I} \frac{dI}{dt} \right). \tag{8}$$

Taking average values, this means that
(9)

$$\frac{dP}{dt}=\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial c_v}\frac{dc_v}{dt}\right)+\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial N_c}\frac{dN_c}{dt}\right)+\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial g_p}\frac{dg_p}{dt}\right)+\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial I}\frac{dI}{dt}\right)$$

since all other terms are zero. If we want the price to stabilize to a static price, we must have that the sequence of values $\left\{\frac{dP}{dt}(t), \frac{dP}{dt}(t+\Delta t_1), \frac{dP}{dt}(t+\Delta t_2), \ldots\right\}$ forms a Cauchy sequence convergent to 0 and which has a finite sum, where Δt_i values represent the intervals at which price changes are made.

A general solution to this problem is that whenever $\left.\frac{dP}{dt}\right|_{k-1}>0$

$$\left.\frac{dP}{dt}\right|_k \leq \frac{1}{(k-l)^\alpha}\left.\frac{dP}{dt}\right|_{k-1} \tag{10}$$

where l is some positive integer and α is some real number such that $\alpha>1$. Whenever $\left.\frac{dP}{dt}\right|_{k-1}<0$

$$\left.\frac{dP}{dt}\right|_k \geq \frac{1}{(k-l)^\alpha}\left.\frac{dP}{dt}\right|_{k-1} \tag{11}$$

where l is some positive integer and α is some real number such that $\alpha>1$.

In particular, this condition is true whenever

$$sgn\left(\left.\frac{dP}{dt}\right|_k\right)=-sgn\left(\left.\frac{dP}{dt}\right|_{k-1}\right) \tag{12}$$

and that

$$\left|\left.\frac{dP}{dt}\right|_k\right| \leq \left|\left.\frac{dP}{dt}\right|_{k-1}\right|. \tag{13}$$

This result is generally true, and does not depend on the particular model that is being employed. Therefore, the result is model-independent, and can be used to design vendors and/or consumer groups whose behaviors will lead to the desired global goal of limited inflation.

3. Examining Agent Models

The minimal condition required for the inflation rate of a simple economic system to settle zero is that the set of price changes forms a Cauchy sequence convergent to 0. While a general solution to this problem is given in equations (10) and (11), we focus on the weaker condition given in equations (12) and (13). Once such a condition is put into place, one may examine the requirements of a system in order to keep it in compliance with this requirement. We examine systems whose design is governed by these equations. Note that we have not yet settled on a model, and the conditions may therefore be considered to be model-independent design criteria.

3.1. **Static costs, population, and no inflation in pricing model.** Let us consider the extremely simple case of a system in which the costs to the vendors are constant, the inflationary rate is not part of the consideration of the price, and the number of consumers entering shops is constant.

$$\frac{dP}{dt}=\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial g_p}\frac{dg_p}{dt}\right), \tag{14}$$

Now, expanding the term $\frac{dg_{p_i}}{dt}$ according to Table 2, we have that

$$\frac{dP}{dt} \geq \frac{1}{N_v} \sum_{i=1}^{N_v} \frac{\partial p_i}{\partial g_{p_i}} \left(\frac{\partial g_{p_i}}{\partial p_i} \frac{dp_i}{dt} + \frac{\partial g_{p_i}}{\partial m} \frac{dm}{dt} + \frac{\partial g_{p_i}}{\partial c_v} \frac{dc_v}{dt} + \frac{\partial g_{p_i}}{\partial pr} \frac{dpr}{dt} + \frac{\partial g_{p_i}}{\partial in} \frac{din}{dt} + \frac{\partial g_{p_i}}{\partial E} \frac{dE}{dt} \right). \tag{15}$$

If we then assume that $\frac{\partial g_{p_i}}{\partial pr} = \frac{dm}{dt} = \frac{din}{dt} = \frac{dE}{dt} = 0$[1], then we are left with

$$\frac{dP}{dt} = \frac{1}{N_v} \sum_{i=1}^{N_v} \left(\frac{\partial p_i}{\partial g_{p_i}} \frac{\partial g_{p_i}}{\partial p_i} \frac{dp_i}{dt} \right). \tag{16}$$

Now, in order to apply the requirement (12), (9) we must have that in cycle k, the new change in price becomes a function of the previous change in price. Moreover, we must have the inequality given in (13). Together this gives us

$$\left.\frac{dP}{dt}\right|_k \geq \frac{1}{N_v} \sum_{i=1}^{N_v} \left(\frac{\partial p_i}{\partial g_{p_i}} \frac{\partial g_{p_i}}{\partial p_i} \left.\frac{dp_i}{dt}\right|_{k-1} \right) \tag{17}$$

where $\left.\frac{dP}{dt}\right|_k$ represents the rate of change of the average price P at iteration k. To simplify the analysis, we can assume that all agents (consumers and vendors) are identical to others from each class. Then, equation (17) becomes

$$\left.\frac{dp}{dt}\right|_k \geq \frac{\partial p}{\partial g_p} \frac{\partial g_p}{\partial p} \left.\frac{dp}{dt}\right|_{k-1}. \tag{18}$$

Thus we must have that $\left|\frac{\partial g_p}{\partial p}\frac{\partial p}{\partial g}\right| < 1$ and $\frac{\partial g_p}{\partial p}\frac{\partial p}{\partial g} \leq 0$. All stable systems subject to the above assumptions must therefore satisfy this relation. The same requirement follows from the case that the r.h.s. is positive in equation (15). Let us consider two sample systems.

Example 3.1. An example of functions (which represent models for vendors and consumers) which have this relation at all points are $p = c_v + kg$ and $g = g_0 - \frac{p}{k}$. In this case $\frac{\partial p}{\partial g} = k$ and $\frac{\partial g_p}{\partial p} = -\frac{1}{k}$. The condition holds for all prices, and therefore, the prices remain stable at all prices.

We can examine a second example for a more complex system.

Example 3.2. Another set of equations, which represents another model of vendors and consumers, might be $p = c_v + kg$ and $g = g_0\left(\frac{c_v}{p}\right)$. Then $\frac{\partial p}{\partial g} = k$ and $\frac{\partial g}{\partial p} = -\frac{g_0 c_v}{p^2}$. This system has an equilibrium price at $p_e = \frac{c_v + \sqrt{c_v^2 + 4c_v k g_0}}{2}$. At this price, $\frac{\partial p}{\partial g} = k$ and $\frac{\partial g}{\partial p} = -\frac{g_0 c_v}{c_v p_e + g_o c_v k}$. Therefore, $\left|\frac{\partial g_p}{\partial p}\frac{\partial p}{\partial g}\right| = \frac{kg_0}{p_e + g_0 k} < 1$ for any equilibrium price, value of g_0, c_v, and k. This means that at the equilibrium price p_e, the swarm condition holds independently of the precise price; $\frac{p_e}{g_0} + k \geq k$ at all prices.

3.2. **Applying the swarm requirement.** In the previous subsection, we examined the requirements of a simple one commodity system consisting of vendors and consumers, each making independent decisions based on their own privately determined criterion. According to our analysis, as long as the requirement that $\left|\frac{\partial g_p}{\partial p}\frac{\partial p}{\partial g}\right| < 1$ and $\frac{\partial g_p}{\partial p}\frac{\partial p}{\partial g} \leq 0$ is maintained, the system's price is stabilized. This should happen no matter the vendor model or the consumer model. This is a

[1] This might happen if the average consumer is not incurring debt and not saving.

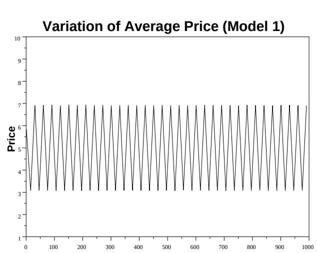
powerful tool in determining how to construct behaviors that generate the vendor and/or consumer models that generate a desired overall economic behavior. We theoretically analyzed two different vendor and consumer models, both of which satisfy these conditions. This analysis indicated that both models would stabilize prices, but one would be selective and the other would not.

This condition can be tested using a simulation of the simple one-commodity economy. We use a simulation consisting of repeated sales interactions between a group of consumers and vendors. Each vendor and each consumer acts independently in each interaction, sharing no information with any other agent. Prior to the first cycle, each vendor determines the price of the commodity. During each cycle, each consumer randomly chooses a vendor to "visit". The consumer determines a "likelihood" that he will make a purchase, given the price of the vendor. This likelihood is a number between 0 and 1 and represents a probability that the interaction will result in a sale. A random number is calculated between 0 and 1. If the value exceeds the consumer's likelihood, then the sale is made. Once all consumers have had a chance to visit all the vendors, the vendors update their prices. Each vendor estimates the demand using the ratio of the number of sales versus the number of visits during the last cycle. This estimate forms the basis for their new price.

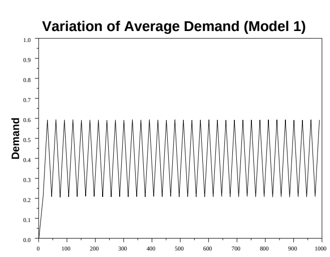
Initially, the vendors' prices are randomly scattered throughout a range of prices which may be deemed from low to high prices. The average price typically begins The program produces a running average price and demand. Since the interactions between the groups of agents are stochastic in nature, the values of the individual quantities can change from iteration to iteration. However, it is the average values that are of interest to us. According to our analysis, these average values should be controlled by the same requirement that controls individual values. Moreover, these are the very global quantities of interest to system designers.

In Figures 3.1 through 3.3, we graph the behaviors of three models of consumer and vendor behavior. The consumer and vendor models used to produce Figure 3.1 have a vendor model given by $p = c_v + kg$ and a consumer model given by $g = g_0 - \frac{2.5p}{2k} + \frac{c_v}{2k}$. The consumer and vendor models used to produce Figure 3.2 are those from Example 1 above. Finally, those used to produce Figure 3.3 are those from Example 2 above. All programs are run with 500 vendors and 100,000 consumers in order to provide a large enough pool to reduce random fluctuations.

Clearly, the consumer model yielding the behavior given in Figure 3.1 cannot satisfy the requirement of (18). Despite this, the behavior of the system is quite dramatic. Initially, the average prices initially start at a mid-range price. However, the average prices rapidly begin to fluctuate. The system settles into a behavior in which the average prices continually fluctuate, and never settle down to an average.



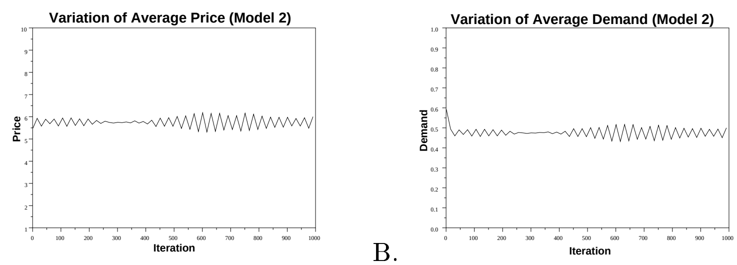
A.



B.

Figure 3.1 The average price and consumer demand for the vendor model $p = c_v + kg$ and consumer model $g = g_0 - \frac{2.5p}{k} + \frac{c_v}{2k}$. The averages fluctuate wildly, with initially small fluctuations settling on relatively large fluctuations.

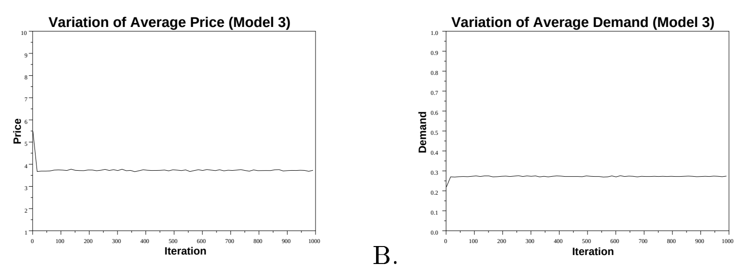
Figure 3.2 represents the average price and demand levels of the consumer models generated by Example 1 of subsection 3.1. These are identical to the models from Figure 3.1 with the exception that the price dependence of the demand is four times smaller than the previous one. This makes the system satisfy equation (18). The average behavior is quite different from that given in Figure 3.1, with the average price and consumer demand rapidly settling to a mid-range value. The stability continues indefinitely once established.



A. B.

Figure 3.2 The average price (A) and consumer demand (B) for the first Example of Section 3.1. The average price and demand, though controlled, still exhibit limited fluctuation.

Figure 3.3 represents the average price and demand levels of the consumer models generated by Example 2 of subsection 3.1. This model is quite different from the other two. The consumer model consists of an adaptive scale in which excessive profit lowers the demand. This model therefore has a mechanism for increasing and lowering the price. Increasing the price enough pushes the model beyond the point where the price is stabilized, yielding a system that is self-stabilizing. The graphs bear this out quite nicely, as they are remarkably smooth and controlled.



A. B.

Figure 3.3 The average price (A) and consumer demand (B) for the second Example of Section 3.1. The average price and demand are strictly controlled and remain virtually static throughout the simulation.

The various average behavior of the three models yields good evidence that our provable condition provides adequate predictive power to conclude that one or the other consumer/vendor model will yield the desired condition.

It is, of course, possible for our analysis to have come from the models directly. We could have generated the vendor and consumer models and then used these models to predict the behavior. However, once the behavior had yielded undesirable behavior, we would have had no intuition about what other consumer models to choose. We would then have had to determine what parameters might change in these behaviors so as to produce the kind of behavior we were looking for. Despite the change from the example of Figure 3.1 to that in Figure 3.2, yielding the much more controlled behavior of Example 1, the behavior in Figure 3.2 is still not optimal. It would have been difficult to generate Example 2, given Example 1 only. Once we had generated it, after much consideration, we could have analyzed its behavior quite handily. It is in the generation of the model that we lack power, and precisely that aspect of the swarm design that gives us the power.

4. Generalizing Models

The models given in Section 3 are definitely simple. The natural thing to assume is that it is one thing to handle a simple one-commodity economy made up of vendors and consumers of only that one commodity economy without worry about inflation, cost increase, money supply, etc., and quite another to handle a complex economy in the real world made up of all these things and more. Moreover, the power of the swarm methodology lies not only in its ability to determine when things are possible, but also in its ability to discern when things are not possible. Finding such a condition allows us to sidestep much work on erroneous economic models trying to understand things that cannot be.

In order to examine different system classes with different conditions, we must begin by generating a set of equations as we did in Section 2. Recalling equation (9), we have that

(19)
$$\frac{dP}{dt} = \frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial c_v}\frac{dc_v}{dt}\right)+\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial N_c}\frac{dN_c}{dt}\right)+\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial g_p}\frac{dg_p}{dt}\right)+\frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial I}\frac{dI}{dt}\right).$$

If we assume that $\frac{dN_c}{dt} = \frac{\partial p}{\partial I} = 0$, we have

$$\frac{dP}{dt} = \frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial c_v}\frac{dc_v}{dt}\right) + \frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial g_p}\frac{dg_p}{dt}\right). \tag{20}$$

Note that we have not assumed that $\frac{dc_v}{dt}$ or $\frac{\partial p}{\partial c_v}$ are zero. This means that the vendors costs and the rate at which these are passed on to consumers are nonzero. In this case, in order to limit the prices of things, the set of equations yields

$$\left.\frac{dP}{dt}\right|_k = \frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p}{\partial c_v}\frac{dc_v}{dt}\right) + \frac{1}{N_v}\sum_{i=1}^{N_v}\left(\frac{\partial p_i}{\partial g_{p_i}}\frac{\partial g_{p_i}}{\partial p_i}\left.\frac{dp_i}{dt}\right|_{k-1}\right). \tag{21}$$

Once again, assuming a uniform set of vendors and a uniform set of consumers, we have that

$$\left.\frac{dp}{dt}\right|_k = \left(\frac{\partial p}{\partial c_v}\frac{dc_v}{dt}\right) + \left(\frac{\partial p}{\partial g_p}\frac{\partial g_{p_i}}{\partial p_i}\left.\frac{dp}{dt}\right|_{k-1}\right). \tag{22}$$

Since the sequence of $\frac{dP}{dt}$ values is a Cauchy sequence, we can guarantee convergence if[2]

$$\left|\left.\frac{dp}{dt}\right|_k\right| \le \left|\left.\frac{dp}{dt}\right|_{k-1}\right| \tag{23}$$

[2] This is the same weaker condition given above. We do not solve the stronger condition here.

and

$$sgn\left(\left.\frac{dp}{dt}\right|_{k}\right) = -sgn\left(\left.\frac{dp}{dt}\right|_{k-1}\right). \tag{24}$$

Again, if $\left.\frac{dp}{dt}\right|_{k-1}$ is increasing, this means that $\left.\frac{dp}{dt}\right|_{k}$ must be decreasing. In this case, using the more general result from (10), we have that

$$\frac{1}{(k-l)^{\alpha}}\left.\frac{dp}{dt}\right|_{k-1} \geq \frac{\partial p}{\partial c_v}\frac{dc_v}{dt} + \frac{\partial p}{\partial g_p}\frac{\partial g_p}{\partial p}\left.\frac{dp}{dt}\right|_{k-1}. \tag{25}$$

Also, we have that

$$\frac{\partial p}{\partial c_v}\frac{dc_v}{dt} + \frac{\partial p}{\partial g_p}\frac{\partial g_p}{\partial p}\left.\frac{dp}{dt}\right|_{k-1} \geq -\left.\frac{dp}{dt}\right|_{k-1}. \tag{26}$$

Together this means that

$$\left(\frac{1}{(k-l)^{\alpha}} - \frac{\partial p}{\partial g_p}\frac{\partial g_p}{\partial p}\right)\left.\frac{dp}{dt}\right|_{k-1} \geq \frac{\partial p}{\partial c_v}\frac{dc_v}{dt} \geq -\left(1 + \frac{\partial p}{\partial g_p}\frac{\partial g_p}{\partial p}\right)\left.\frac{dp}{dt}\right|_{k-1}. \tag{27}$$

This equation essentially says that the quantity $\frac{\partial p}{\partial c_v}\frac{dc_v}{dt}$ is sandwiched between two vanishing quantities. Recall that $\left\{\left.\frac{dp}{dt}\right|_{k}\right\}_{k=1}^{\infty}$ is a Cauchy sequence convergent to zero. This means that the limit is

(28)

$$-\lim_{k\to\infty}\left(\frac{1}{(k-l)^{\alpha}} - \frac{\partial p}{\partial g_p}\frac{\partial g_p}{\partial p}\right)\left.\frac{dp}{dt}\right|_{k-1} \geq \frac{\partial p}{\partial c_v}\frac{dc_v}{dt} \geq -\lim_{k\to\infty}\left(1 + \frac{\partial p}{\partial g_p}\frac{\partial g_p}{\partial p}\right)\left.\frac{dp}{dt}\right|_{k-1}$$

or

$$0 \geq \frac{\partial p}{\partial c_v}\frac{dc_v}{dt} \geq 0. \tag{29}$$

In order for this to be possible, the rate of increase of vendor costs must be zero. However, this would contradict our assumption that $\frac{dc_v}{dt} > 0$. Therefore the two conditions are inconsistent[3]. Thus, it is impossible to hold prices steady with any measurable positive rate of change of vendor costs *using any model of consumer behavior.* Moreover, *it is impossible to hold prices steady using any model of vendor behavior unless* $\frac{\partial p}{\partial c_v} = 0$.

This last result is a stunning result. What makes it so surprising is the way in which it was reached. It makes intuitive sense, yet without the definitive proof in hand, the sneaking suspicion that some behavior somehow defies this conventional wisdom exists. It is somehow satisfying to prove that the intuition is not intuition alone, but rather a statement of rigorous proof.

5. Directions

Swarm engineering is now entering a relatively mature state with respect to its state since its inception more than ten years ago. At that time, the general question revolved around robotic systems. The original question for robotic systems was whether or not it was possible to determine the characteristics of a swarm that would accomplish a given task simply from the statement of the task. This swarm

[3]Incidentally, prior to fully understanding what equation (27) was saying, we built a simulation to test an update model for g. In each case, g converged to zero. The result stymied us until we realized that this was the only way consumers could satisfy the requirement.

design would extend to the number of castes of robots and the characteristics of each caste. Each caste behaviors, sensors, and requisite computational capability would naturally result from the design. Moreover, we wanted a system that would provably design the swarm of robots so that it was not necessary to build and test, refine and retest, and potentially rebuild over and over to generate the desired swarm. A great deal of progress has been made on this approach to swarm design. The methodology that has arisen has been applied successfully to many robotic systems (though all have been simulated)[1] and to thermodynamic systems whose properties were designed using swarm methodologies[2].

The basic starting point for swarm engineering design is a listing of all measurables *that are available to the various agents for inspection and potential reaction.* This would include, in the present system, the prices of things, the number of consumers, and others. This would not include quantities such as GDP, exact number of pieces sold in the market, total income of the consumers, or other quantities. Though estimates might be provided, the actual number is not known or available to any of the agents and so does not form part of their decision-making process. Thus, these quantities only should be used to construct the global goal, which might be one of the quantities that isn't available to the agents. The reason is that their derivatives represent actions of the agents, and they are things that the agents can control. Using a property designed in this way gives access to the designers of the system to the very things that the agents can do, and allows one to make strict statements about required behaviors which lead to the desired global goal.

In the present work, we extend the swarm methodologies to economic systems, along the same lines as [3]. Our general assumption is that individual consumers and vendors act like autonomous agents, and in that sense, may be modeled as swarms. We have indicated the various quantities available to the individuals in our system, and also worked through their differential relations. The relations between the different quantities would seem to be the defining qualities of the systems we call economic systems, and many of these cannot be predicted as they are subject to the discretion of the individual agents. Using these, however, we can begin to create expressions for the conditions under which certain outcomes may be expected to occur. Once these expressions have been understood, we can generate agents that satisfy them knowing that the desired outcome will be the natural result of the system. Moreover, any agent model which satisfies these conditions will provably generate the desired system-wide behavior.

In this paper, we have applied this approach to very simple economic systems consisting of a single commodity. In both cases, we have attempted to determine a general condition for the vendor and consumer agents which will yield a stable set of prices. In one case, we derived a model-independent condition which, once satisfied, would generate stability. In the other case, we derived an expression for a requirement for stability, and then reasoned that this condition could not ever be satisfied. In both cases, the results were general, and not limited by assumptions concerning agent behavior or interactions.

The methodology represents a novel approach to the design of economic systems. Rather than concentrating on working out ever more precise and relevant models of consumer or business behavior and using these to generate group behaviors, the approach uses the group behaviors to generate the consumer or business behaviors. If the consumers or businesses conform to these conditions, the global outcome will follow. Getting the consumers and/or businesses to actually behave in this way would seem to be a separate issue. However, the method has the ability to make real provable statements about global outcomes that depend only on meeting a behavioral goal. It would seem that using this, it would be possible to determine if particular economic systems are feasible, derive the design of the systems, and quickly test them in simulation once the system has been designed. Future use of this approach may allow the design of new economic systems along with requirements for the behavioral models needed to put them into place.

References

[1] S. Kazadi, J. Lee, J. Lee. *Artificial Physics, Swarm Engineering, and the Hamiltonian Method.* **Proceedings, World Congress on Engineering and Computer Science 2007**, San Francisco, California, USA, October 24-26, p. 623-632, 2007.

[2] S. Kazadi and M. Webb. *Swarm Pumping.* Working Paper 3, Jisan Research Institute, 2006.

[3] S. Kazadi, P. Kim, J. Lee, J. Lee. *Swarm Economics.* **Proceedings, World Congress on Engineering and Computer Science 2007** , San Francisco, California, USA, October 24-26, p. 602-610, 2007.

[4] L. Tesfatsion. *Agent-Based Computational Economics.* ISU Economics Working Paper No. 1, Revised July 2002.

[5] S. Yu and J. Yuan. (2005) *Agent-Based Computational Economics: Methodology and Its Application in Electricity Market Research.* **Proceedings, The 7th International Power Engineering Conference (IPEC)**, Singapore, Nov. 29 2005-Dec. 2 2005.

[6] J. Yuan, W. Ding, Z. Hu. *A Critical Study Of Agent Based Computational Economics And Its Application In Research Of Electricity Market Theory.* **Power System Technology**, 29(7), pp.37-41, 2005.

[7] L. Tesfatsion. *Introduction to the special issue on agent-based computational economics.* **Journal of Economic Dynamics and Control**, 25, pp.281- 293, 2001.