Identification of Shapes Using A Nonlinear Dynamic System

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Abstract

We describe a nonlinear system capable of use as a recognition system. This system is composed of a set of coupled oscillators, connected by linear springs. Images are overlaid on the system by altering the masses and spring constants of the oscillators, thereby modifying the detailed behavior of the system. Signatures extracted from the system using FFT of the individual oscillator positions show coherance, relative continuity, and translational and rotational invariance. These properties are discussed in the context of the eventual use of this system as a general identification system. **keywords**: nonlinear dynamic system, pattern recognition

Introduction

Historically, visual object detection has tended to fall into one of two basic methodologies, template matching and approaches based on image pattern relationships, such as light intensity (Papageorgiou, 1997). One of these methodologies is the set of methods which depend on gross properties of images such as the intensity distribution as a function of wavelength or the overall average luminosity. Another is *generalized template matching* in which small features are extracted from the image in question, and their positional relationships are used to identify the image. These features can correspond both to features representing physical structure and to features representing regional variations.

Characteristically, systems dependant on relationships view images as a whole rather than a combination of the individual parts which comprise it. Such a process generates a single value for a given characteristic. In the classification of lumber, for example, it may be advantageous to detect wood-type by the brightness of a sample. By using a "training set" of lumber to generate a frequency distribution for brightness of a given variety of lumber, unknown samples can be classified based on correlations with these distributions (Duda and Hart, 1973).

Contrary to relationship systems, template matching employs algorithms to partition images into desireable regions. The positions of these individual portions relative to one another are the basis for recognition. In cases when it is only necessary to locate objects rather than classify them, such as the problem of finding humans in a given scene, template matching is often used. Once many parts of an object have been detected from the same region in a picture, it is assumed that the object in question is present in that area (Mohan, 1999).

A third process created as an extension of standard template mathing was found by attempting recognition through the use of "template ratios," whereby a human face is represented by the relationships between the average intensities of specified regions, such that changes in illumination were not of significance (Sinha, 1994). Although this proved relatively successfull, it suffers from the same problem as all other template systems. The characteristics by which regions are located are not based on any rigorous scientific or mathematical foundation. Rather, these regions are found through pre-determined assumptions of the user. Therefore, the reliability of such a system is limited to the accuracy of the human-defined regions.

We investigate in this paper the use of a highly nonlinear system as an object recognition system. This system uses the system's inherant structure to generate a single signature, which itself will be a unique representation of the system's structure. It is very much like both methods, and may be viewed as a combination of both, as it is capable of incorporating features of the system into an overall measurement.

The choice of a nonlinear system is not arbitrary. It stems from some of the limitations inherant in the use of linear systems. The chief attractive feature of a linear system is the linear independence. Linear independence is concisely expressed by the superposition principle which basically states that

$$O\left(\sum_i \vec{x_i}\right) = \sum_i O\left(\vec{x_i}\right) \tag{1}$$

for some operator O and some set of vectors $\{\vec{x_i}\}$. The use of the superposition principle enables a much easier analysis of a linear system than that of a non-linear system. If a system has N inputs, the system's response can be determined by adding the individual responses of the system due to each input. Also, multiplication of an input by a scalar number will result in a scalar multiplication of the response by the same number. Linear systems work well when inputs are independent of one another, and there are no interactions between them. However, if the inputs are not independent, and there are interactions between inputs, the system response is nonlinear, and linear systems are not well suited in this case. In image recognition problems, the pixels which make up the image can be thought of as a set of inputs. Since the patterns in the image represents relationships among pixels, the inputs are not expected to be independent of one another.

Non-linear systems are capable of producing complex behavior. This complex behavior may seem random at first, but when they are viewed in phase space, they can display very high level complex patterns. The patterns produced by a non-linear system are contained in an attractor. An attractor may be defined as the set of trajectories traced by the system's states as the system evolve under the influence of the system's dynamics. The most interesting features of an attractor are:

1. attractors are bounded,
2. there can be infinite number of trajectories in an attractor,
3. and small differences in the initial conditions produce very different trajectories.

Although the trajectories in the attractor are sensitive to the initial conditions, the attractor themselves are remarkably consistent. For a given system, the attractor is the the same regardless of the initial condition.

For image recognition problems, the relationships among the pixels can be modeled as the interactions among inputs. The interactions among inputs will create non-linear dynamics which, under certain circumstances, will generate an attractor. Since the attractor is unique for a given dynamics, the dynamics is the result of the interactions among the inputs which represents the relationships among the pixels, the relationships are defined by the pattern in the image, the attractor can be used to identify the pattern in the image.

Inasmuch as an attractor represents its generating system, one may think of using attractors to identify the system it generates. In this paper, we present the use of chaotic attractors in the identification of images. We generate dynamic systems using a system of coupled linear oscillators whose properties are generated in part by the image. This system produces a chaotic attractor, and the properties of this attractor may be used to identify the image.

1 Theory

1.1 Measurements

The use of nonlinear dynamic systems requires an understanding of the underlying dynamics of the system at such a level as to produce a useful property or set of properties. Our goal is to understand chaotic attractors at such a level as to be able to utilize their properties in the identification of images. We therefore begin with definitions clarifying these properties. We assume that the reader is familiar with the concept of phase space, and take this to be the launching point of our work.

Suppose that we have a system S defined by a set of n degrees of freedom $\{x_1, ..., x_n\}$. We can represent this system as a point in the n-dimensional real space. Thus, we may represent the system as $\overrightarrow{x} \in \Re^n$. The use of the time-derivative gives the complete state of the system, and we may think of a system as its combined position and time derivative of the position. This gives us a system description such as $\left(\vec{x}, \frac{d\vec{x}}{dt}\right)$. From this, we may define an attractor.

Given our system S and its equations of motion and constraints, the set of points in phase space corresponding to allowed configurations of the system are set. We may denote this set as Γ and define it as

$$\Gamma = \{\vec{x} \mid S(\vec{x}) \text{ is allowed}\}$$

The set of all allowed initial points to a trajectory are the set of all allowed points which are not exclusively the final points in trajectories. That is, any trajectory may be thought of as a set of initial points, which each start the trajectory consisting of those points which follow the point. These will not include the points that make up point attractors. Thus,

$$I = \{\vec{x} \in \Gamma \mid \vec{x} \text{ is not an attractor of the system}\}.$$

Then we may define a propagator of the system. Given the dynamic equations of motion, there exists a propagator P_S which maps the set of all initial points in phase space to their corresponding points in phase space at later times. This propagator acts on the initial point $\vec{x}_0$ and maps it into another point $P_S(\vec{x}_0, t) = \vec{x}_t$ where t represents some time in the future after the time at which the system was at $\vec{x}_0$.

A ***trajectory*** $\tau_{S,\vec{x}_0}$ is defined as the set of all points arising from the action of the system on points in phase space. More formally,

$$\tau_{S,\vec{x}_0} = \{\vec{x} \mid \exists\, t \in \Re \,.\ni .\, \vec{x} = P(\vec{x}_0, t)\} \ .$$

We say that a trajectory is ***bounded*** if there exists an ϵ-ball $B(\vec{x}; \epsilon)$ which contains the trajectory.

From these trajectories arise attractors. As a stable system evolves trajectories in phase space, these trajectories will tend to clump, forming a region in space, known as an attractor. This region typically has a well defined multidimensional shape and extent. We may define it as follows:

$$A = \{\tau_{S,\vec{x}_0} \mid \vec{x}_0 \in I\} \ .$$

we define the attractor as stable and bounded if there exists an $\epsilon > 0$ and an $\vec{x}'$ such that $A \subseteq B(\vec{x}'; \epsilon)$.

Each attractor is defined by the trajectories that define it. However, it may be the case that each trajectory is unique. Thus, a single trajectory cannot define the attractor. On the other hand, each trajectory may be viewed as a sampling of the attractor that creates it, and it can be easily shown that any attractor will come arbitrarily close to each point off the attractor.

Given a particular trajectory in phase space, one may have a number of different measurements which depend only on that trajectory. The most notable one, of course is the length of the trajectory. This is given by

$$M_l = \oint_\tau d\overrightarrow{s} \qquad (2)$$

where $\vec{s}$ is a unit vector along the length of the trajectory τ. However, one may imagine other measurements which are defined by the trajectory as well. These measurements are defined by

$$M_f = \oint_{\tau} f(\vec{s})\, d\vec{s} \tag{3}$$

where f may or may not be a vector function of the position in the trajectory.

Empirically, it has been found that many measurements done on different trajectories in chaotic systems are identical. Such a property is extremely useful if the system is dependant on the image. This means that different instances of the same system should be expected to generate the same measurement despite differences in initial conditions. This we take to be a necessary property of any chaotic system to eventually be used in an object recognition system. This, of course, comes out of the fact that many chaotic systems are indeed ergodic, and thus different trajectories will have similar statistical properties.

At present, it is does not seem to be known under which conditions chaotic systems become ergodic. We defer this discussion to another paper. However, we present a dynamical chaotic system which we believe to be ergodic. This system may be used, as we shall show, to identify different images used to modify the basic system.

1.2 Clustering of Measurements

We have already commented on the ergodicity of the system we intend to describe. Such ergodicity gives us a way of identifying the system despite the detailed initial conditions, and despite the later deviations of trajectories in phase space. However, an ergodic system with such an invariance property cannot be used to differentiate between different objects. This is because every image will tend to have only minor differences in the system behavior, making identification impossible. Thus, the use of a single invariant system in order to generate the measures that yield identification would not seem to be fruitful.

It is well known that different systems of the same type have the capability of exhibiting very significant changes due to a change in the detailed parameters of the system. This change can happen due to a very small change in the detailed parameters describing the system. Our system, although chaotic, is quite different from this. In our system, changes in the system cause commensurate changes in the measure, though these changes occur continuously. Thus, the system empirically seems to represent a continuous mapping between image space and signature space, with various invariences becoming consequences of this mapping. This is known as *clustering*, and it allows a simple threshold to be used in determining the identity of the system.

This is a very desirable state of affairs in image recognition, when objects of the same class are clustered in the same group in signature space. That is, two oak trees will tend to be more alike than, say, two ginko trees. By the same token, it is desirable if two people are clustered together far from two lions, which are wholly different. This same state of affairs may allow one to group large numbers of pictures of the same person in the same area of signature space, allowing the identification of the same person through many different stages of life and allowing uninterrupted identification.

More rigorously, we report work on a system that represents a two step procedure. The first step consists of a mapping of images onto a system in such a way that they create chaotic dynamics. Thus

$$c : I \rightarrow A \tag{4}$$

where I is the set of images, and A is the set of attractors to which it is mapped. Moreover, we develop a measurement M which maps the set of attractors to a signature space

$$M : A \rightarrow S \tag{5}$$

in which the desired properties occur. Thus, our desired mapping is

$$M \circ c : I \rightarrow S\ . \tag{6}$$

In the next section, we describe a system in which the mapping appears to behave in this way.

2 Incorporating an image in a dynamic nonlinear system

We are interested in studying the construction of a nonlinear system with the properties outlined above. Most importantly, we wish to develop a system which exhibits basic characteristics of a continuous mapping between picture and signature space. This means that, given a $\delta > 0$, $\exists$ a resolution and a positive number ϵ at which a sufficiently small alteration to the picture may be made which will cause the signatures of any given pictures differing by less than ϵ to have a distance smaller than δ. Moreover, basic desired properties of the system include scale invarience, as well as translational and rotational invariance of a given image in a given visual field.

We use a toroidal lattice of linear oscillators as the basic element in our system, depicted in Figure 2.1.

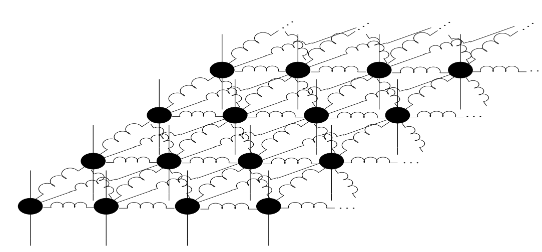


Figure 2.1: A lattice of coupled oscillators. Each oscillator oscillates in the z-coordinate, and is connected to eight neighboring oscillators via springs.

Each node contains a single oscillator, whose oscillation is constrained to the axis perpendicular to that of the toroid. Each node corresponds to a single image pixel. This means that an image of size $n \times m$ requires an identically-sized lattice of nodes. The nodes are located in Euclidean plane. Each node is connected by springs to adjacent nodes, yielding eight neighbors per node. Nodes are equidistant from their neighbors. Nodes on edges are connected to nodes on the opposite edges, completing the toroid.

Pictures may be overlaid on the system of oscillators by choosing a chatacteristic in the picture and using this characteristic to modify the properties of the oscillators. The property under modification may range from the mass of the oscillator to its spring characteristics. We consider black and white pictures, whose pixels are either on or off. Pixels that are on correspond to oscillators with higher masses and different spring constants than those that are off. This allows the system's dynamics to reflect the image.

The system is initialized in a state in which all oscillators are motionless and located at their equilibrium point, with the exception of the oscillators corresponding to image elements. These latter pixels are pulled to a height above their equilibrium point commensurate with some characteristic in the picture – in our case whether or not the pixel is turned on, as we use only black and white images. In order to allow the system to produce consistent signatures, the oscillators are run for some measure of time. This removes any measurements that might depend on a transient response of the system produced as a result of starting from rest. Once the system has reached a steady state, the positions of each of the oscillators is recorded for a predetermined number of iterations. Each sequence of positions if subjected to a fast fourier transform (FFT). The logarithm of all resulting FFTs are summed, producing the signature of the image. This signature may then be compared with those from other images.

It is worth noting, in view of the previous theory, that each step in this process occurs via a continuous transformation. Only one process, that of summing the individual FFTs may be seen to be a possible discontinuous action, whose discontinuity might arise from the discreteness of an individual space. It may be possible to demonstrate that in the limit of infinite resolution, the fourier transforms themselves are continually varying across the image, and so summing them represents a multidimensional integral of continuous functions. In this limit, the mapping between image space and signature space may be seen to be continuous. Although we consider this to be plausible, we do not assert that it is true. Rather, we view it as an important component of future research.

3 Pictures and Signatures

The above procedure was applied to several different data sets, of which representative data sets will be presented. The data falls into five distinct categories: that which demonstrates the uniqueness of pictures of different classes, invariance in the identification of different size pictures in the same class, invariance in the identification of pictures with different orientations, invariance in the identification of pictures of the same class at different positions , and continuity between two different objects. We present the data, and discuss its implications as to the nature of the system. We defer to the next section a discussion of the importance of these properties in terms of identification systems.

3.1 Translational Invariance

The design of our system implies that translations of identical images should produce identical signatures. At any position on the lattice, the system will be identical, differing by a simple rotation compared to the original system. This will tend to produce images that differ at most by rounding errors. This is desireable in a recognition system.

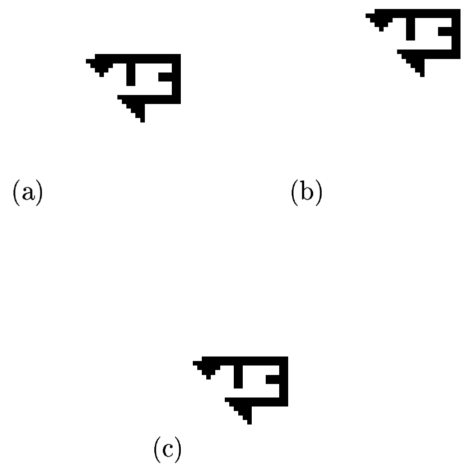
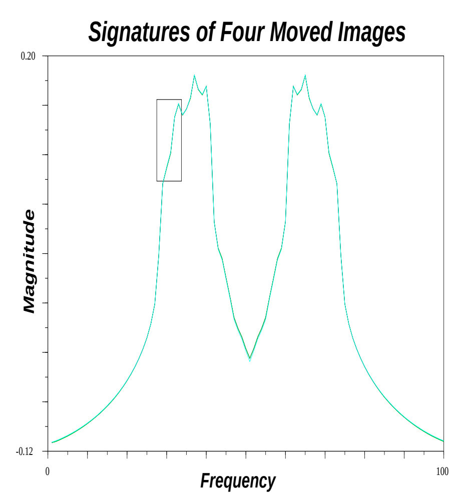
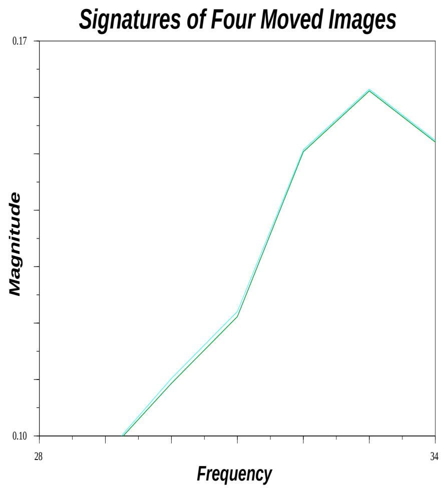


Figure 3.1: These pictures are typical of images used to test the translational invariance. The pictures are moved across the lattice, yet the final signatures are nearly identical.

We utilize several sets of data, of which one representative set is presented. Each of the objects in these images is displaced relative to other images. As expected, in Figure 3.2, we find almost perfect overlap of the signatures. Only a very small variation in the zoomed graph is apparent.



(a)

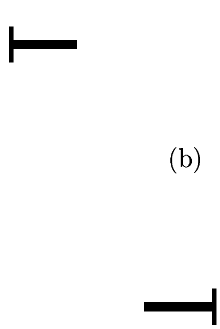


(b)

Figure 3.2: These graphs depict the signatures of the system created by translating a simple image across the grid. As expected, the images nearly overlap.

3.2 Rotational Invariance

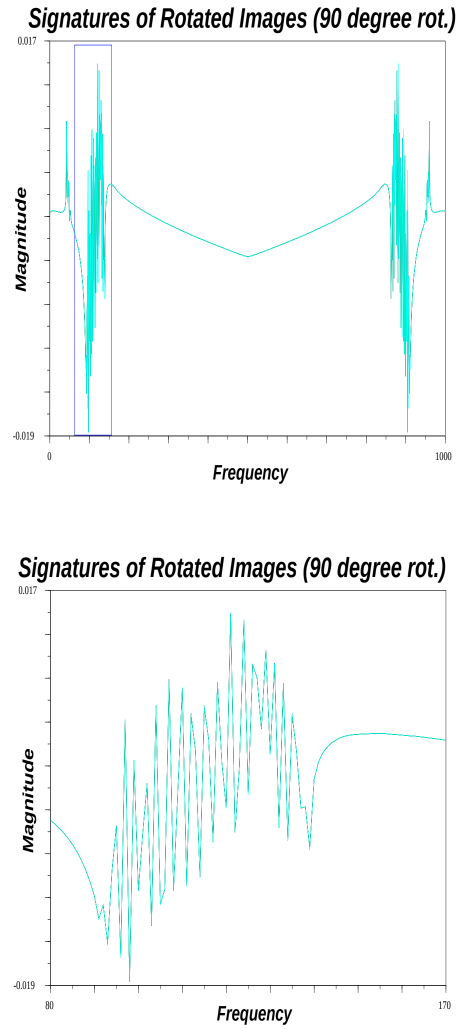
The system has obvious rotational symmetry. That is, an image imposed on the system will interact with an identical grid despite any rotation that occurs. This again is an extremely advantageous property. This may be illustrated by using images rotated by $\frac{\pi}{2}$, as in Figure 3.3.



(a) (b) (c)

Figure 3.3: These pictures illustrate the rotation of an object found on our grid.

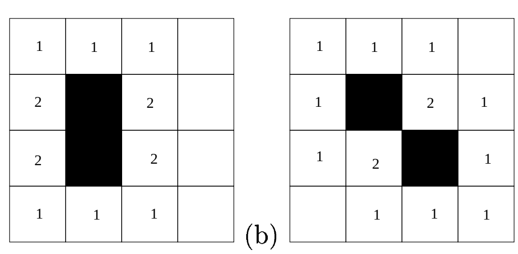
Figure 3.4 illustrates the overlap of the signatures created from these images. Note that the signatures are so alike, that only differences due to rounding errors are detected.



(a) (b)

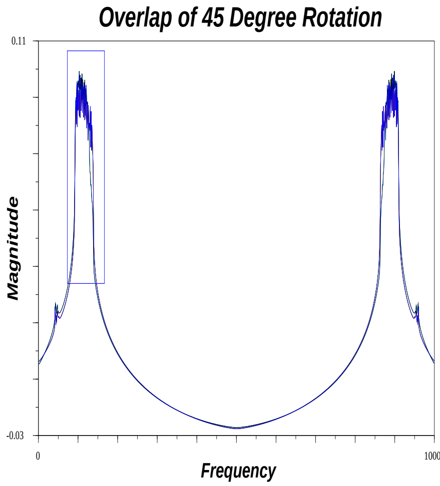
Figure 3.4: These signatures result from the rotated images depicted in Figure 3.5. The overlapping signatures confirms the rotational invariance of the system.

One difficulty with the system arises from the particular geometry of the system. As illustrated in Figure 3.5, the neighborhood structure of a simple pair of pixels changes dramatically if the pixels are rotated by $\frac{\pi}{4}$. This causes significant differences in the signatures, as in Figure 3.6.

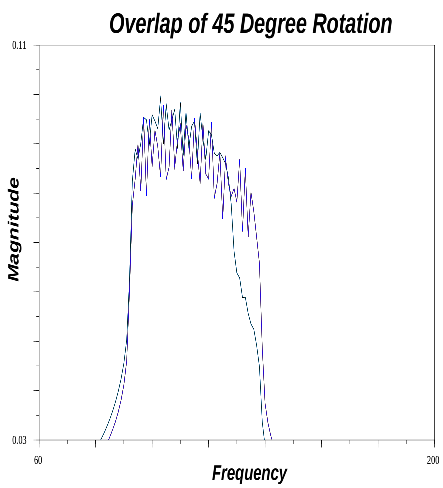


(a) (b)

Figure 3.5: The neighborhood structure changes when the image is rotated. In these images, a dark square indicates a pixel which is turned on, while the numbers indicate how many activated pixels the square is adjacent to.



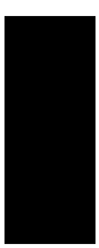
(a)

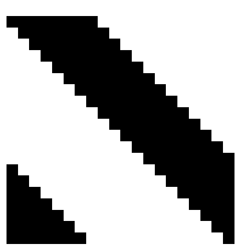


(b)

Figure 3.6: The signatures of rotated images are significantly different at rather low resolution.

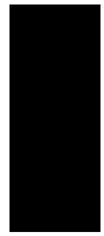
Low resolution is the key, however. At higher resolutions, the differences tend to disappear. This indicates that in the high resolution limit, we have true rotational invariance.

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

Figure 3.7: These images depict the same object in two rotational states in different resolutions. The higher resolution images would seem to differ less significantly from their counterparts than the lower resolution images.

Practical use of such a lattice would seem to require one to determine the difference resulting from such a rotation, and to verify that different images have sufficiently different signatures to distinguish them from simple rotations.

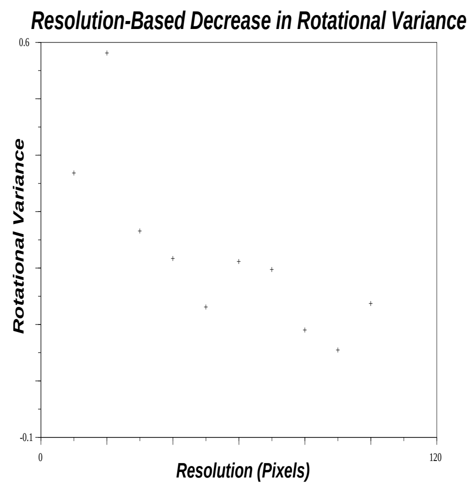


Figure 3.8: This figure illustrates the reduction in differences between rotated images at different resolutions. Clearly, high resolution creates significantly smaller variations.

Although Figure 3.8 illustrates that increases in image resolution do not lead monotonically to improved rotational invariance in the signature, as the resolution is increased, the system tends to allow for greater freedom of rotation without variation in signatures. Thus we expect rotational invariance to be found, as asserted, at high resolution.

3.3 Uniqueness

The foundation of any visual recognition system is some form of uniqueness. This establishes the ability to distinguish between different objects. For uniquness to exist in our system, the attractors generated by two pictures must be distinctively different. This means that the time-variant behaviors of differing images should be significantly different, allowing the signature of the pictures to have evident differences. In order to demonstrate show this property, we plot the signatures of four different pictures in Figure 3.9. As is evident, the signatures are significantly deviant, and may easily be identified as different.

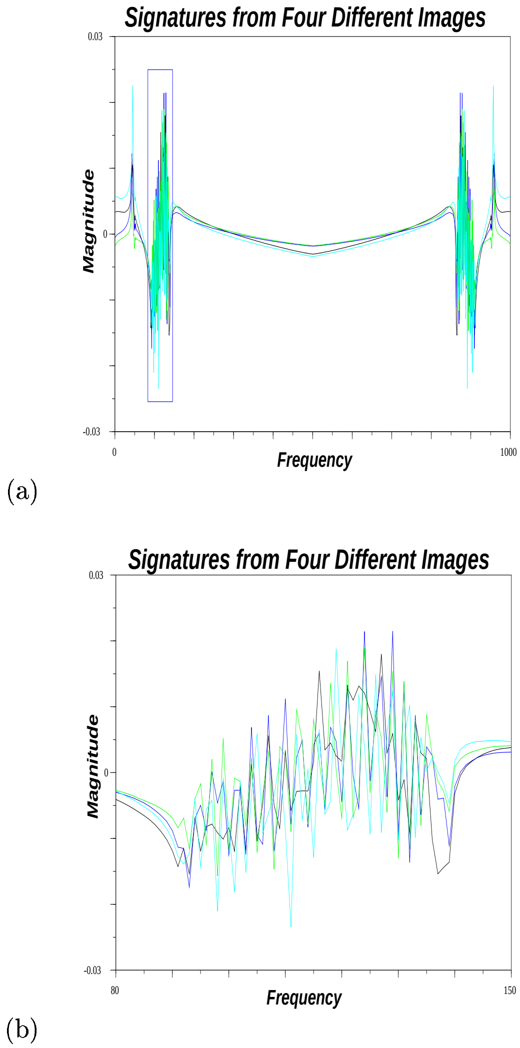


Figure 3.9: Signatures from four different images. Each pair of similar images produces a pair of similar signatures, which are distinct from one another. The level of distinction is sufficient to allow rudimentary identification.

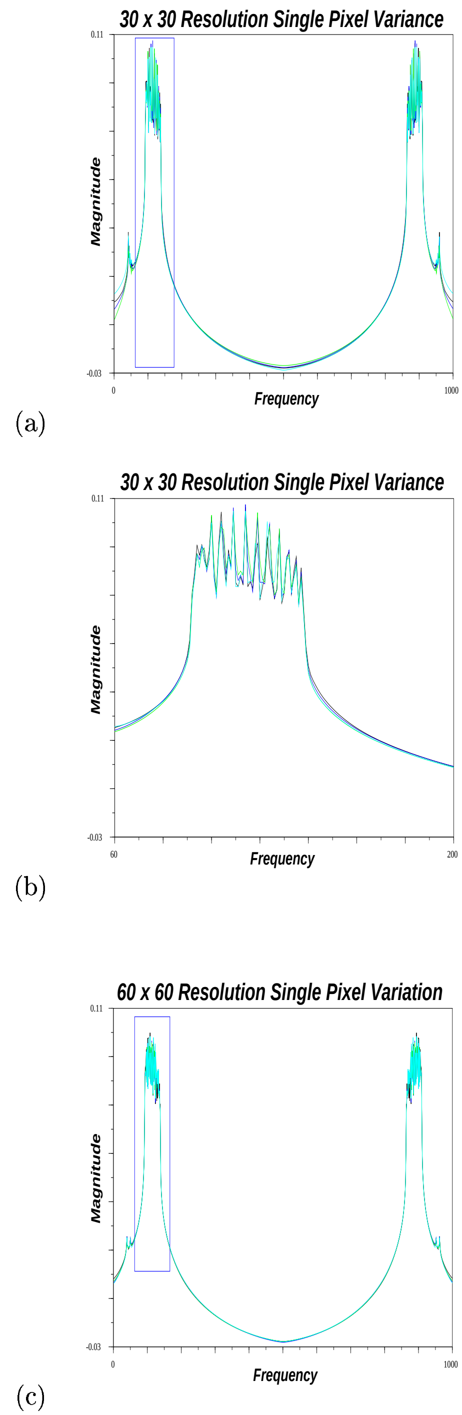
3.4 Continuity

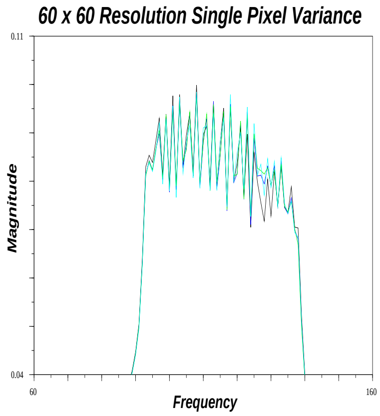
In examining continuity we cannot make claims of an $\epsilon - \delta$ nature. Images are made up of a specific resolution, making the determination of a given δ for any specific ϵ impossible. In order to make claims about continuity, we must therefore view δ as a minimum resolution, and rewrite the definition of continuity using this.

Definition: We define an ***infinite resolution continuous mapping*** ψ between image space and signature space, assumed to be a normed vector space with norm $||_s$, to be one such that given any $\epsilon > 0$, $\exists\, \delta > 0$ such that if the images have a *resolution* of δ and if $|I - I'|_i < \delta$, then $|\psi(I) - \psi(I')|_s < \epsilon$.

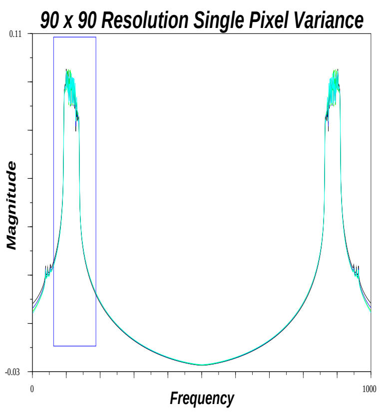
Note that any two images, the distance between which is δ or must have a resolution of δ or less. In the absence of this condition, it is not possible to have two different images whose difference is less than δ. Using this definition of an infinite resolution continuous mapping between the image space and the signature space, we can talk about whether or not the mapping between image space is indeed continuous. As the mathematical proof of this property is beyond the scope of this study, we demonstrate empirically that the system appears to behave in a way consistent with the existence of a continous mapping.

We accomplish this by investigating the effects of random changes to a given image at different resolutions. Random variations are introduced by altering individual bits and calculating new signatures. We present representative data at resolutions of 30×30, 60×60, and 90×90 in Figure 3.10. In this figure, it is evident that single changes, expected also to be representative of multiple changes, generally decrease in relative magnitude as the resolution increases.

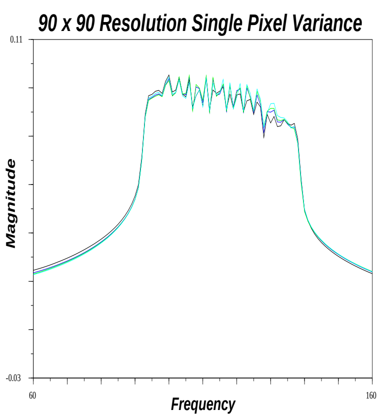




(d)



(e)

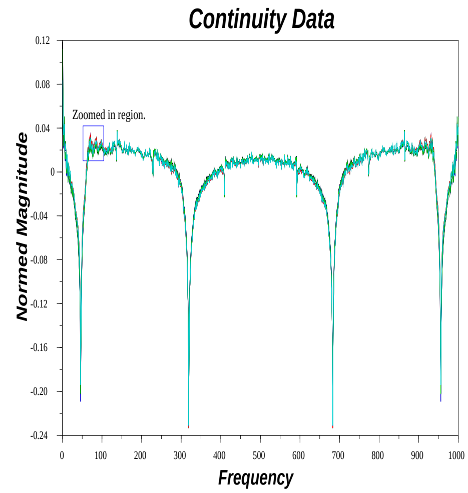


(f)

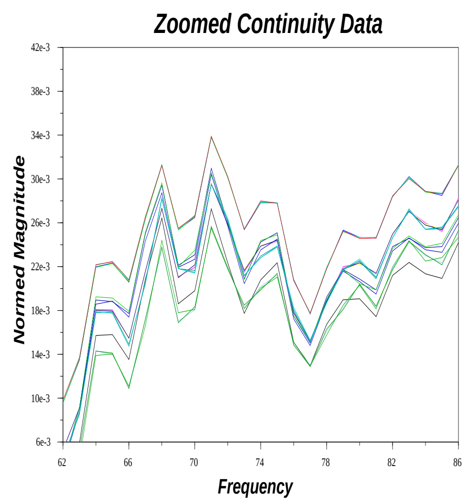
Figure 3.10: As the resolution increases, there is a concommittant decrease in the variance among signatures created by similar images differing by single pixels.

Interestingly, this mapping does not seem to be uniformly continuous. This means that different alterations would seem to cause different changes in the signature. This may be related to why the increase of resolution does not produce a monotonic decrease in rotational variance. Despite this phenomonon, we have yet to observe any case that contradicts that the mapping is indeed an infinite dimensional continuous mapping. In the absence of such data, we may state that this mapping, and therefore this system, satisfies the final condition, and await theoretical verification.

Indeed, in Figure 3.11, we plot several different signatures generated from similar images.



(a)



(b)

Figure 3.11: These images show the signatures of several different images of the same general design. As one can see, thought these do not overlap, the images are quite similar.

These data support our assertion of continuity in our system, though as discussed above, this is not true $\epsilon - \delta$ continuity.

4 Discussion and Conclusions

Many recent visual recognition systems have typically been designed around a specific class of objects of interest. This requires one to build feature detectors which find specific predetermined features in a particular object. For instance, one may find eyes, noses, mouths, ears, eyebrows, etc. when finding a face. However, a face is not made up entirely by the simple features found within it. The tilt of the eyes, the cheekbones, indentations on the cheeks, length of the forehead, etc. all contribute to the overall understanding one obtains when viewing a face.

We have developed a system that takes a first step in this direction. Continued work in this direction may lead to the development of a somewhat different method of performing identification. Rather than requiring the scientist to determine what makes up the important set of features found within the picture, the entire picture may be used in its identification. This means that although the eyes and nose and mouth would be important constituents of the image, they are by no means the only constituents. Indeed, though we are a far distance from this desirable goal, we have been able to demonstrate some of the features one might want to have in such a system.

Firstly, our system demonstrates some measure of rotational invarience and true translational invarience, with rotational invariance subject to the constraints of the system's resolution. At high resolution, the variation due to rotations is expected to become negligible. This expectation is well supported by our data, though a theoretical verification of this fact is outside the scope of this paper. This particular characteristic is useful if the object or objects of interest may not be placed in the same orientation with respect to our sensors. This might be true if the objects are not fixed, say as a person approaching an ATM, or if the sensor is not fixed, say as a camera mounted on an aircraft. Second, the use of translational variation allows us to recognize the object without requiring it to be in the center of a visual field. Such a characteristic is also advantageous in a number of applications in which the field of view is expected to be larger than the object.

Continuity is a tricky subject in discrete object recognition. It cannot be demonstrated empirically that the system obeys continuity. At best, we can demonstrate that the sequence of differences in characteristic signature is Cauchy as the resolution increases. This would also seem to indicate that variations in a picture of a given resolution would produce signatures of a given maximum difference, with this maximum depending heavily on the magnitude of the change in the picture. Thus, as long as the changes in the picture were small, we would be able to create an acceptable difference in the magnitude of the change signature, and be able to identify the object.

Despite the fact that this system is promising, there are still several problems to be overcome before it might be used to implement real object recognition. The first problem arises from the true rotational invariance we would like to have in the system. As discussed above, this would seem to be only approximately possible, due to the difference in neighborhood structure upon changing the orientation of an image. However, possible improvements in the design of the lattice itself and its connectivity may reduce the impact of rotations. For instance one might utilize a hexagonal lattice rather than a rectangular lattice, and the maximum rotational variation would seem to come about at a rotation of 30 degrees rather than 45 degrees.

Other problems include ways in which to achieve true scale invariance. The design of a scale invariant system might require some form of fractal design for the lattice, which would allow the image to have the same design despite its scale. This would seem to work if the image was truly fractal, but of course could only approximately be accomplished using real discrete lattice components. Careful design of such a system would seem to be an interesting future research direction.

While there are many challenges to the creation of practical systems based on this design, the possible uses of this work are relatively wide open. Of particular interest would be the use of this system in any kind of security system including ATM systems, door lock systems, etc. Uses of large numbers of these systems functioning in parallel would seem to provide sufficient individuality to the different images to be able to be used in real security systems.

Other uses of this technology are also more practical. As this might be able to be used to identify different regions of space and landmarks, this might provide an extremely low computation method of determining precise locations as part of a topographic map in autonomous robots. Moreover, very detailed identification of the visual scene might be possible, allowing very precise behavioral switching necessary for applications such as swarm engineering (Kazadi, 2000) which would require simple agents to be able to understand their environment, despite rather stringent computational cost requirements. This technology might provide a cost effective, lightweight alternative to time consuming computations based on high resolution cameras and rather quick processors.

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