A ROBUST CENTRALIZED LINEAR SPATIAL SEARCH FLOCK

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ABSTRACT

In this paper, we explore the development of a robust robot chain designed to allow non-pheromone-mediated target localization and transportation to a central location. The method is based on the development of a lossless robot swarm capable of carrying out a search of a local area in such a way that each individual robot is capable of determining the direction along the chain that one might follow to the center of search. We investigate some of the stability issues of the swarm, examining the stability during the setup phase, after the setup phase, and during catastrophic losses of individuals in the chain. We also explore potential methods of using the mechanism to deal with obstacles and multi-resource exploitation.

KEY WORDS

swarm engineering, flocking, distributed search

1 Introduction

Flocking is a behavior commonly found in nature in which organisms position themselves into close groups and maintain sensory contact with one another. Variations on this basic behavior abound in the animal Kingdom, with animals ranging from fish to insects to land mammals to birds participating in one or more methods of flocking. The behaviors in the animal world seem to be largely centered around generating a competitive advantage, though the great variety of ways in which animals can generate advantageous group behaviors is impressive.

Recently, many researchers have spent much effort on the development of coordinated control of multiple vehicles. Such results center around the development of controlled convoys or semi-dynamic groups of individual agents capable of moving together in a coordinated way. While this work is capable of making rather accurate predictions about the behavior of groups of individual agents moving in unison from an initial state, it is not clear how this can be applied to groups of agents in which the number of agents and their instantaneous states are not immediately predictable. As an example, these systems do not typically take into account the potential for individual agent failure or erroneous behavior. As a result, such systems are somewhat less adaptable than one might desire.

On the other hand, there is a great deal of work on flocking, with the emphasis on using adaptive pairwise interactions which together determine the group behavior. In much of this work, the group behavior is somewhat unpredictable, as a result of ad hoc behavioral protocols. Using physical robots, many researchers demonstrate flocking behavior in constrained laboratory environments. The dynamics of such systems are surely different from flocking behaviors in unconstrained or unpredictable environments, and it is unclear how generalizable this work is to a real flocking application in which individual agents have limited global information and no barriers to continual unbounded motion. However, such a situation is surely to be used for practical applications, and it is important to understand how flocking algorithms must be designed in order to deal with unbounded space.

Recent work on puck clustering work [3, 4] has indicated that clustering systems may be able to be used as precursors to distributed construction tasks involving swarms of robots. These missions require the agents to be able to carry out complex construction projects, though the individual agents need not have detailed knowledge about the construction itself. One practical problem that agents have in generating construction tasks is the development of resource localization and transportation systems. These are mechanisms by which agents are used to locate sources of materials, as they become available, and transport these materials to a central location, such as a construction site. Much work on mechanisms based on the behavior of ants [5, 6, 7] has been done in recent years, with the main contributions coming from the use of pheromone-mediated mechanisms. The difficulty with such a mechanism is that in realistic applications, the use of stationary markers or volatile chemical markers is unrealistic; less wasteful methods might provide a more cost effective swarm. Moreover, it is relatively unlikely to find a swarm protocol based on such methodologies that provides hard upper limits to the detection of resources as they become available. Finally, there is slim chance that the methods that have been devised to date are capable of being implemented on truly autonomous mobile agents, as their ability to stay in the swarm, rather than wandering off, is very much dependent on their ability to determine their position and that of other agents and/or landmarks. Removing this capability would have unpredictable effects on the swarm.

In this paper, we begin by investigating the last of these issues. We examine the global requirements for an absolutely cohesive swarm of autonomous agents *bereft of a global information provider*. Section 2 illustrates that a coordination mechanism is a necessity when designing cohesive swarms, and that pairwise interactions are insufficient to provide cohesion. In Section 3, we present the design of a swarm of pairwise reactive robots arranged in a chain with a global coordination mechanism based on pairwise interactions. Section 4 examines various properties of the swarm of robots including the time to assemble the chain or robots and robustness to failure of the mechanism. Finally, Section 5 gives some concluding remarks and potential future areas of research.

2 Minimal Conditions for Lossless Flocking

Several investigators have explored the development of distributed methods of generating coherent flocks of individuals. This work has appeared in various forms in various publications. For the most part, the swarms have been either confined to a specific localized area, particularly in the realm of real robotic swarms, or have exhibited movement in a single direction and handled obstacles as they appeared. The methods have, in general, failed to deal with mechanisms required for distributed and coordinated search in unconstrained areas requiring the lossless maintenance of flocks using only local information. Thus, we examine this now as a way of determining the minimal condition for obtaining coordinated search in an unconstrained environment.

There are a number of methodologies for holding swarms of individual agents together in one coherent group. These vary from the local neighborhood-based potential methodologies to those requiring detailed knowledge of the position of a given agent with respect to other agents and/or positions in the search area. In general, the idea of *coherence* can be summarized as follows. Each agent position may be defined by a vector $\overrightarrow{x}$ in n-dimensional space (which is typically two- or three-dimensional). In realistic simulations, the agent's velocity may be given by $\dot{\overrightarrow{x}} \equiv \overrightarrow{p}$, with the speed varying between 0 and some p_{max}, which is dependent on the agent's physical limitations. Finally, the acceleration varies between some maximal acceleration u_{max} and 0 and may be denoted by $\dot{\overrightarrow{p}} \equiv \overrightarrow{u}$.

In order to maintain coherence, it must be the case that at all times,

$$|\overrightarrow{x_i} - \overrightarrow{x_j}| \leq S \tag{1}$$

for some maximal swarm size S. Moreover, we assume that the control mechanisms are provided by reactions to locally sensed agents; agents outside of some maximal sensory range d_s cannot be sensed. We desire, in general, some way of determining a minimal condition for the application of control which guarantees that an initially coherent swarm will not separate into an incoherent swarm.

In general, an equilibrium is maintained if a restorative "force" is applied to any system deviating from some desired state. In other words, it is possible to establish a global state or set of states of the system which may be defined by a state vector $\overrightarrow{s_A} \in A \subset R^n$ which is the desired state. Deviation from this state induces a transition to a state vector in a larger set B which contains A, and for which a restorative action is carried out in such a way that all elements evolve in time to elements of A.

We define a measurable on A to be an operator O such that $O : R^n \mapsto \Re$. A consistent operator is one in which $O\overrightarrow{s_A} < O\overrightarrow{s_B}$ for all elements $\overrightarrow{s_A}$ of A and elements $\overrightarrow{s_B}$ of B. Let us suppose that the basis for A can be defined by $\{b_1, \ldots, b_n\}$. Moreover, let A_i be the singleton set consisting of only element i in the basis. Then, clearly,

$$A = \langle A_1 \rangle \otimes \langle A_2 \rangle \otimes \cdots \otimes \langle A_n \rangle \tag{2}$$

Let the vector $\overrightarrow{v_{A_i}} = (0, \ldots, 0, v_i, 0, \ldots, 0)$ be the restriction of v to the subset A_i. Finally, we define a local operator to be one in which the operator maps to the linear span of a subset of basis elements. That is, the operator O_l carries out

$$O_l : \langle A_n \rangle \otimes \cdots \otimes \langle A_{n+m} \rangle \mapsto R. \tag{3}$$

Let us consider the deviation of the ith individual from the other elements in the swarm. We can represent this as

$$\tilde{\overrightarrow{x_i}} = (\overrightarrow{x_1} - \overrightarrow{x_i}, \overrightarrow{x_2} - \overrightarrow{x_i}, \ldots, \overrightarrow{x_k} - \overrightarrow{x_i})$$
$$\tilde{\overrightarrow{p_i}} = (\overrightarrow{p_1} - \overrightarrow{p_i}, \overrightarrow{p_2} - \overrightarrow{p_i}, \ldots, \overrightarrow{p_k} - \overrightarrow{p_i}).$$

Then, in general, our control algorithms have the form

$$\tilde{\overrightarrow{u_i}} = C_i \left[O_{i1}(v_1, p_1), \ldots, O_{ik}(v_k, p_k) \right]. \tag{4}$$

The minimal condition for coherence is that $\tilde{\overrightarrow{u_i}}$ provides a restorative force such that $|\overrightarrow{x_i} - \overrightarrow{x_j}| \leq S$, or $\tilde{\overrightarrow{x_i}} \rightarrow (0, 0, \ldots, 0)$. If we let $i = 1$ for simplicity, (4) becomes

$$\frac{\partial^2 \overrightarrow{x_1}}{\partial t^2} = C_1 \left[O_{11}(v_1, p_1), \ldots, O_{1k}(v_k, p_k) \right]. \tag{5}$$

But, as $O_{ij} = 0$ for all $|\overrightarrow{x_i} - \overrightarrow{x_j}| > D$ where D is the sensory distance of an individual in the swarm, this cannot be a simple pairwise action. In this case, without a loss of generality only the first m swarm members are sufficiently close for sensing. Thus, the elements beyond m are reduced to a zero contribution to the control mechanism. Thus, $\tilde{\overrightarrow{u_i}}$ becomes

$$\frac{\partial^2 \tilde{\overrightarrow{x_1}}}{\partial t^2} = C_1 \left[O_{11}(v_1, p_1), \ldots, O_{1m}(v_m, p_m), 0, \ldots, 0 \right]. \tag{6}$$

This cannot, in a general reactive system, be guaranteed to yield an appropriate restorative force that maps $\tilde{\overrightarrow{x_i}}$ to $(0, 0, \ldots, 0)$ unless a method of coordinating the entire swarm's actions is utilized. This mechanism may be called a *coordination mechanism* and defines how the various swarm individuals are linked to one another. It is important to note that this mechanism is a global mechanism, and thus defines the minimal condition for the coherence of a swarm of individuals.

In many flocking mechanisms, the coordination mechanism is ignored. Pairwise interactions are created describing how the swarm elements respond to one another. However, in the absence of a global coordination mechanism, the swarm can break up into multiple subsets, which is undesirable, particularly in a swarm-based search algorithm. As a result, care must be taken to generate a coordination mechanism which has the global effect of limiting the spread of the swarm and of providing a restorative behavior in the presence of the growth of the swarm.

Several researchers have built retrograde coordination mechanisms as a result of their pairwise interactions. In these studies [1, 2], a Hamiltonian description of the energy of a configuration is generated, and from this, appropriate pairwise equations of motion are generated. The description includes a pairwise potential function which has a minimum at a particular desired separation between elements of the swarm. This mechanism can be viewed as a coordination mechanism, as long as the potential is strong enough to overcome any inertia of a subset of the whole swarm. If not, it is conceivable for a subset to move far enough away from the main group that it is capable of "breaking" a connection to the whole swarm, thereby severing the tie between the swarm and the group. The strength of the potential is thereby viewed as the coordination mechanism and is capable of holding swarms together robustly, but only of a size which depends on the potential itself. If the potential's strength is insufficient to hold the swarm together, this mechanism fails because it is incapable of globally coordinating the swarm.

In view of this requirement for lossless flocking, we describe our swarm of agents in terms of not only their pairwise interactions but also their coordination mechanism. The coordination mechanism employed here will guarantee that the swarm does not lose individual agents.

3 Chain-Based Swarm

In this Section, we introduce a methodology for maintaining a coherent swarm of agents arranged in a chain capable of performing a local search. This work is similar to that of Fredslund and Mataric [8] which uses "friendship chains," though in that method, each agent was not generally interchangeable. In our system, the search is mediated by pairwise communication and local sensing that implements a global coordination mechanism. The method is capable of building stable chains of agents whose properties depend on the parameters of the method and on the number of agents.

3.1 Agent Behaviors

Each agent is assumed to be autonomous in the sense that it does not require a global controller or information from the outside world to decide its action at any moment of time. The agents act and react based on information stored in memory and conditions that are capable of being sensed locally. In the case of our agents, only two numerical memory elements are required, making the controller of the agent particularly simple and minimalist. Moreover, all sensory capabilities of the robot may be based on simple obstacle avoidance and low power beacons. The devices used by these agents might be fabricated for autonomous robots from materials costing well under \$100.00, if being produced en masse.

The two numerical elements that each agent contains store the agent's current state and determine how the agent will react to sensory information. For simplicity, we label these elements the state and the spiral memory. Moreover, each agent has a transmitter which broadcasts the state of one of these memory elements. The transmitter is unidirectional and is assumed to be able to be used to identify the direction from which the signal originated. Thus, each agent is capable of communicating one numerical piece of data to other agents in such a way that they may use this information to determine the location of the transmitting agent. All agents are assumed to be moving at all times with the exception of a single "leader" agent. Each agent decides, based on sensed information and the current state, how to change its direction of travel.

The state memory defines the placement of agent in the chain of agents. Agents initially have a state given by -1 which indicates that the agent is not in a chain, and thus should either search for a chain to join or the end of a located chain. Agents with a state value of 0 do not move; other agents will form a chain starting with this agent. This agent serves as the global beginning of the chain, and this state represents the primary part of the global coordination method. Only one 0 element can exist per chain. An agent with a state given by an unique positive number is in the chain with its position indicated by the state. This helps the agent keep its position in the chain, reacting to changes in the positions of other agents appropriately as they happen.

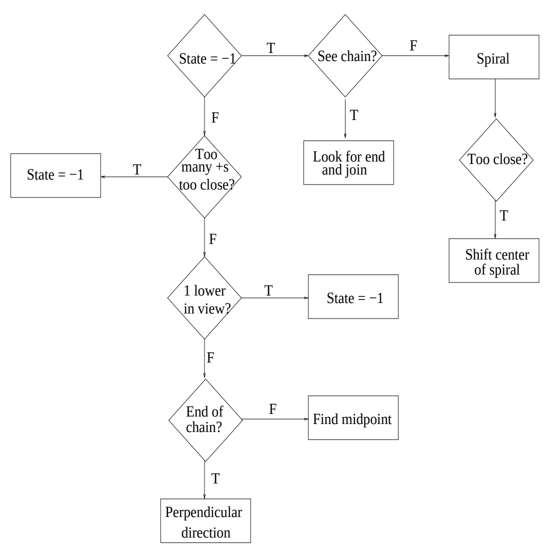

Figure 1: This diagram gives the low-level behavior of the robot.

Initially, all agents are in the -1 state. This means that all agents are in a search mode, looking for a chain. The search consists of an increasing spiral which is meant to cover the local area. The spiral behavior is mediated by the second memory location, which tracks the time the robot has spent in a spiral. Individuals encountering one another along the way execute random turns and continue the spiral from the current position, effectively moving the center of spiral. Each agent continues this behavior until it encounters a chain of agents.

An agent in the -1 state encountering a chain will move directly toward the agent with the highest state. Upon approaching very near the highest state agent, the agent with the -1 state will attempt to orbit the highest-state agent in a counter-clockwise direction. This orbiting will continue until the agent finds itself at the end of the chain, along the same line that the last two agents in the chain are on unless the agent becomes of the state 1. Adding to a chain, of course, occurs with some error. This error will become important later when considering how long and stable a chain can be. An agent's spiral timer will be reset upon the first encounter with a chain so that unsuccessful additions to the chain lead to initially small spirals.

Agents in the positive states have two behaviors. First, those agents in the interior of the chain move directly toward the midpoint of the line segment joining the agents whose states differ by one from the moving agent. If an agent determines that it is at the end of the chain (i.e. there are no larger states visible to the agent), its direction of motion varies between a direction perpendicular to the ray joining it and the next lower-state agent and the direction that brings it back to the next lower-state agent. This has the global effect of moving the entire swarm in a counter-clockwise direction, sweeping out a circle or radius approximately equal to the length of the chain.

Finally, an agent in the positive state will return to the -1 state if it cannot any longer see an individual with a state one smaller than the current state.

The entire behavior is diagrammed in Figure 1.

3.2 Chain Dynamics

Each of the behaviors described in Section 3.1 is individual behavior governed by the internal state of the individual agent and the sensory input of the agent. It is of interest to us to examine the initially expected global behavior in the face of the local behaviors. We begin by examining the global coordination method used in this design.

As discussed in Section 2, having some method of holding the flock together which carries global information is important in order to have a static and stable flock, despite the fact that some of that information[1] may be mediated by local mechanisms. In our case, the global information is tied up in the direction one must move to reach one end or another, along with information about the distance to the head of the flock. The coordination method, then, is defined by its ability to hold the robots together which is mediated by the movement and specificity of the individual's position in the chain, as well as the specificity of choosing the position in the flock.

The second part of the coordination mechanism is a method of guaranteeing that the flock remains coherent even in the face of perturbations. Aside from the movements of the individual agents which tend to keep them in the proper spot in the flock, the agents have a secondary mechanism that returns them to the flock once it has been lost. This mechanism is the spiraling mechanism. The spiraling mechanism serves to keep the swarm centered around the same general area as time progresses so that collisions become a significant problem. It also allows the swarm to recapture the flock once it has been lost. This means that the swarm tends to remain in the same general place as time progresses, even when it has not yet completely formed the chain.

4 Chain Flocks

In this Section, we illustrate the dynamics of the chain-flocking behavior by presenting some representative flocks and describing issues related to the convergence time and stability of the flock. All simulations are run with the agents initially randomly placed within a space of size 200 ft. x 200 ft. (each robot has a radius of 1 ft). The initial state is as in Figure 2.

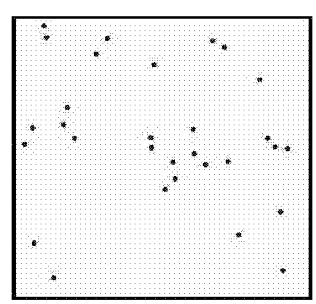

Figure 2: This figure illustrates the initial state of the system of agents.

4.1 Chain Formation

As this is a swarm of individual agents containing no central controller, the formation of coherent structures is interesting in that it seems to behave almost as though there were a central controller. Let us describe a typical chain formation as illustrated in Figure 3.

[1]Note that in some cases, this information is not quantitatively observable from every part of the swarm. For instance, if each element of the swarm were connected to the last one by a potential, then the global information can include whether or not an individual is oscillating, rather than the details of the position of the individual. Nonetheless, this is global information concerning the entire swarm.

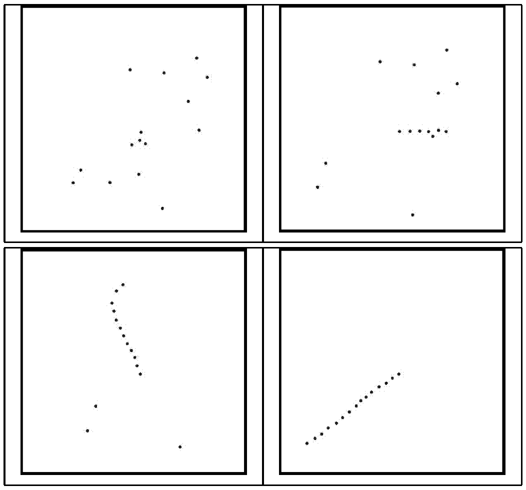

Figure 3: This figure illustrates the formation of a chain of agents starting in an initially randomly distributed configuratoin. This configuration quickly attracts the robots that are closest to the state 0 agent, gradually picking up more robots.

As the simulation starts, one of the agents randomly transitions from state -1 to state 0. This may happen multiple times, though the probability is so low that the likelihood of its happening twice is very small. As each of the agents adds itself to the chain, the chain begins to coherently swing around the agent in state 0. This swinging of the chain moves the chain alternatively out of range and then back into range of the agents initially far away from the chain. As these agents add themselves to the chain, the reach of the chain grows and the more outlying agents can be collected. A typical chain formation is illustrated in Figure 3.

The chain formation progresses rather quickly at the outset and happens more slowly as time continues. This is mainly because the swarm of agents disperses rather completely due to collisions and random repositioning of spiral centers. Thus, those agents that are further out from the state 0 agent tend to join the chain only after their spiraling behavior brings them into contact with the chain.

Each agent in the chain joins the chain with a predetermined angular deviation allowed. Deviations greater than this do not result in the adoption of the state, which is necessary for the growth of the chain. Because the deviation is nonzero, this means that the chain need not be completely straight. (In our simulations, a deviation of 2° is the maximum allowed.) In general, the chain is not straight, but bends in a coherent way. This means that, rather than a straight line, the chain loops over, with the looping causing a destabilization in the chain, as shown in Figure 4.

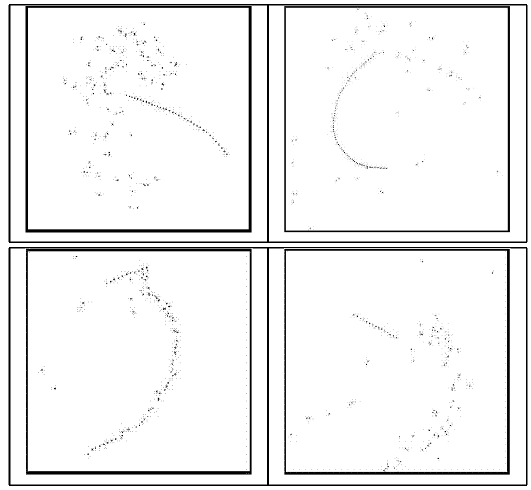

Figure 4: Adding too many agents to the chain can cause the chain to loop over and destabilize.

This destabilization happens when too many individual agents come together and end up executing an obstacle avoidance behavior, which causes the remainder of the chain to transition to individuals in state -1. These individuals tend to alternatively reform the chain and collapse the chain. As a result, the chain has a maximum allowed size after which it cannot sustain any new agents.

It is interesting to ask how the time to form the chain increases with the chain length. A knowledge of this may give us information about how long one might expect a real system of this type to take to set up. We graph data relating the chain length to chain formation time in Figure 5.

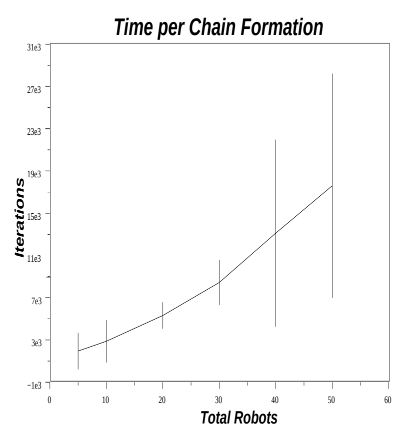

Figure 5: The chain formation time increases approximately linearly with chain length.

In this figure, the formation time is nearly linear up to the near maximal chain size. Thus, as the chain grows, we may predict the time it takes to form.

Finally, the chain sweeps around at an angular speed which is approximately inversely related to the length of the chain, particularly at short chain lengths. This is because the speed is controlled by the outermost agent. The outermost agent moves along at a speed which is constant, but in a direction that is perpendicular to the chain at the point it is located. However, as the chain length increases, the speed deviates from the inverse relation, as the chain curves with increasing prominence. Because of this curving, the sweep speed deviates significantly from the inverse relation. We graph this in Figure 6.

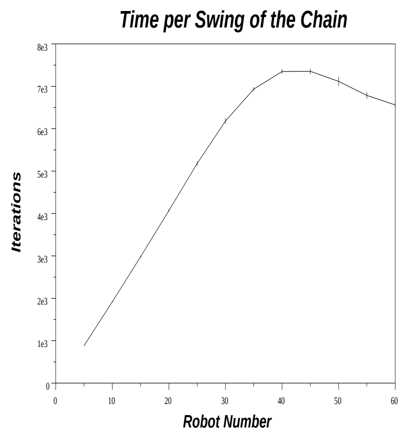

Figure 6: The angular sweeping speed varies inversely with the radius at low chain lengths, but deviates significantly from this at larger lengths.

The predictability of the angular speed is very significant because it allows the chain formation to be used with high reliability. Systems designed in this way which are made for automatic deployment and organization can be used with very high reliability. This is especially important in the case of low tolerance for coverage or target recognition failures.

4.2 Agent Failure

One of the most important benefits of swarms is the redundancy and its ability to influence systems' robustness. Since no individual agent is specifically required in order for the swarm to function, one or more agents should be able to be deactivated without the catastrophic loss of functionality. That is, any system built using the system of robots should be able to survive the collapse of an individual. How the system survives is of interest both in terms of the ability of the system to reconstitute itself and in terms of the time such a reconstitution should take.

In the case of our system, the loss of a single agent has a number of immediate consequences. Immediately, the agent in the chain furthest from the state 0 agent and closest to the failed agent will recognize that it is at the end of the chain. The chain now rotates with this agent setting the speed. All other agents will fail to see a preceding agent in the chain, forcing a transition to the -1 state. This produces a searching behavior for the chain, after which the chain will reconstitute without the deactivated agent. This situation is depicted in Figure 7.

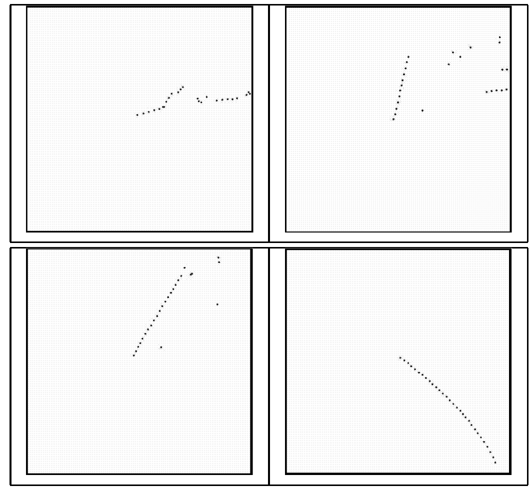

Figure 7: These images illustrate the collapse and reconstitution of the chain of agents in response to an agent failure.

The ability of the swarm to reconstitute can be inferred from the capability of the coordination method to initially constitute the chain.

It is also interesting to ask how long such a reconstitution will take in comparison to the initial formation. Naturally, longer time than the initial constitution is expected because of the length of the chain in comparison to the original distribution of individuals. In Figure 8, we plot the reconstitution time as a function of the number of agents.

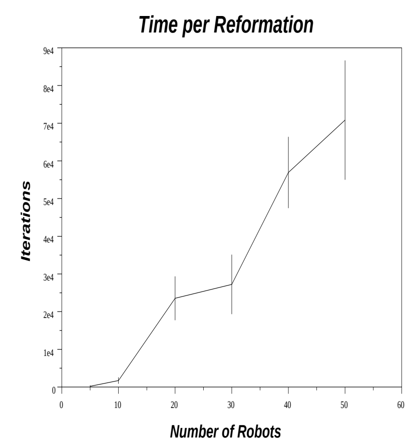

Figure 8: This image illustrates the time required for reconstitution as a function of the number of agents upon failure of the fifth robot in the chain.

Clearly, the reconstitution time increases as the number of agents increases. Interestingly, the reconstitution time is still largely linearly related to the number of agents, as was the constitution time.

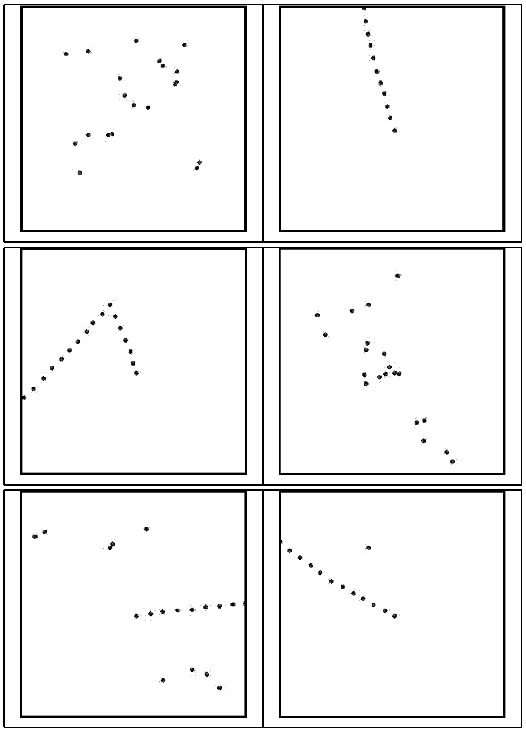

Figure 9: These images illustrate the failure of a single agent's movement, and the subsequent evolution of the system due to the collapse. The single agent causes the "folding" of the chain, which then deconstructs and reconstructs without the crippled agent.

Although not catastrophically, a similar situation can occur if an agent fails. With real robots, the most common failure is mechanical, as in the failure of a wheel or a drive train. Remarkably, such a failure will cause the collapse of the chain with a probability approaching one as time increases. The resulting reconstitution will include the failed agent only with a very small probability, which effectively excises the agent from the swarm. If the failures happens at the end of the chain, then simple addition of more agents to the system can compensate for such an event.

5 Discussion and Concluding Remarks

The development of coherent swarms of agents is a very important topic of research, particularly when discussing the use of swarms capable of exploring unbounded areas and reacting to changing abundances of materials. Of perhaps the most importance is the development of methods of guaranteeing the coherence of a swarm of agents in such a way that no agent is lost unless it is damaged. No research results addressing this particular question for mobile reactive robots is known to the authors of this study. In this paper, we have theoretically illustrated the need for such a mechanism, which we call a *coordination mechanism*.

We have then described such a system made up of pairwise reactive agents which seems capable of forming with complete reliability into a robust chain of agents capable of searching a predetermined area in a reliable amount of time. The system appears to be remarkably robust, retaining all of the agents during the original organization, and reorganizing automatically as agents experience complete or partial failure. Moreover, the system has several predictable characteristics such as the time required for formation, the global angular speed of the swarm, and the length of the maximum stable chain.

This system illustrates the potential robustness of a system of agents in the face of uncertainty as a result of a coordination mechanism that provably keeps the swarm intact[2]. Thus, it can be concluded that such a mechanism is an important part of swarm design, and must be done carefully.

The system illustrated here might be used for a variety of potential interesting applications. The dynamics of such a system when more than one state 0 agent emerge are an interesting topic to explore. How will the chains interact with one another? One might expect that the chains would attenuate according to their interactions, limiting the size of one or the other based on the existence or lack thereof of other chains of greater length at the interaction point. One might also ask how this might be used, for instance, to create a self-arranging and repairing system of independently sweeping detectors. Such a system might be used in volatile situations requiring the continued input of new agents and removal of existing agents. This could be used as a method of deploying, in an unsupervised and very quick way, a large number of agents designed to cover and surveil a large area. Finally, one might ask how this system might be modified to deal with obstacles so as to build chains of agents that are capable of reaching resources that are far away and located among obstacles.

6 Acknowledgements

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