Variance in Converging Puck Cluster Sizes

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ABSTRACT

We investigate the design of control algorithms for puck clustering simulations. Of interest is the control of the variance in puck cluster sizes, particularly when multiple clusters are being created. We present a theoretical framework under which behaviors may be designed which serve to control puck cluster sizes.

keywords: swarm engineering, puck clustering

1. INTRODUCTION

Puck clustering is a field of inquiry which is a subset of a much larger field of inquiry known as swarm engineering. Swarm engineering is the use of simple agents to accomplish complex tasks. The premise is that the capabilities of the swarm exceed the individual capabilities, allowing a great number of tasks to be efficiently carried out which would have been either impossible or excessively time consuming for an individual agent of even moderate intelligence.

To date, very little work has been done on examining the mathematical framework of clustering, although a great deal of work has been done on examining the mechanisms behind both biological and robotic clustering [1][2][3][4][7][8]. For the most part, these studies examine the microscopic behaviors either employed by natural systems or designed by human designers to carry out clustering. While these provide tantalizing examples of the successful ways in which swarm engineering may be utilized, they do not provide a theoretical framework under which the behavior of the system may be controlled simply from designing the behavior of the agents according to some predetermined methodology.

This paper extends previous studies in puck clustering [5][6] which indicate that the development of the system may be correctly predicted by utilizing theoretical results to derive the design of the behavior of the agents. Those results, which were largely theoretical in nature, demonstrated the general conditions for clustering, how to discriminate between methods of differing efficiency, and how to create multiple clusters of predetermined size ratios. This paper examines the method of controlling the variance when creating multiple clusters. We are interested in understanding the theoretical design of individual agents that will create the desired group behavior.

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2. SUPPRESSION OF VARIANCE

Understanding that clusters of equal sizes can be generated [5], we begin our investigation of the variance seen in multiple clusters by considering a system of two clusters and the dynamics which produce identical sized clusters. Without proof, we assert that our results may be generalized to systems of more than two clusters. Let n_i equal the number of pucks in cluster i. Let n_{ie} equal the number of pucks at equilibrium, which is the average size of the clusters when generating equal sized clusters. Let r_t equal the total number of robots present, and r_c be the number of robots carrying pucks. f is probability of a robot removing a puck from a cluster as a function of the cluster size, and h is the probability of adding a puck. The rate of change of the number of pucks in the cluster is:

$$\frac{dn_i}{dt} = -(r_t - r_c)f(n_i) + (r_c)h(n_i) \tag{1}$$

We approximate the above using Taylor polynomials:

$$\frac{dn_i}{dt} = -(r_t - r_c)[f(n_{ie}) + (n_i - n_{1e})f'(n_{ie})] + (\mathrm{r}_c)[h(n_{ie}) + (n_i - n_e)h'(n_{ie})] \tag{2}$$

Rearranging we obtain:

$$\frac{dn_i}{dt} = [-(r_t - r_c)((n_i - n_{ie})f'(n_{ie})) + (r_c)((n_i - n_{ie})h'(n_{ie}))] + [\text{-}(\mathrm{r}_t - r_c)f(n_{ie}) + (r_c)h(n_{ie})] \tag{3}$$

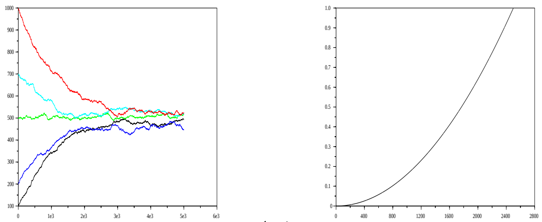
Since n_e represents the cluster at equilibrium, and the rate of change at equilibrium is 0, the equation can be simplified to:

$$\frac{dn_i}{dt} = (n_i - n_{ie})[-(r_t - r_c)f'(n_{ie}) + (r_c)h'(n_{ie})] \tag{4}$$

The last equations is a linear equation with restoring constant equal to $-(r_t - r_c)f'(n_{ie}) + (r_c)h'(n_{ie})$. Therefore, to increase the restoring constant it is necessary to increase the magnitude of this constant by increasing the first derivatives of f and h at the predetermined equilibrium point.

3. APPLICATION OF THEORY

We can form clusters of equal sizes by using an increasing puck removal probability function as in Figure 3.1B.



(A) (B)

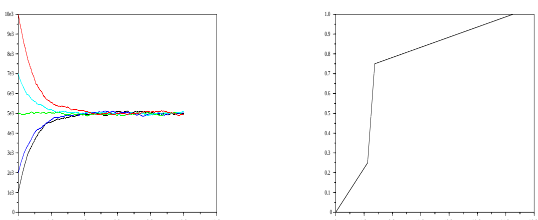
Figure 3.1: non-embodied puck clustering simulation in which clusters evolve to the same size.

We can optimize the function used. Lower variances should be able to be obtained by utilizing functions with higher first derivatives at the equilibrium point. We may examine this using:

$$f(n) = \begin{cases} \frac{n-(e-500)}{2000} + \frac{1}{2} & \text{if } (e-500) < n < (e+500) \\ \frac{n}{4((e-500))} & \text{if } n \leq (e-500) \\ \frac{n-(e+500)}{4(n_t-(e+500))} & \text{if } n \geq (e+500) \end{cases} \quad (5)$$

$$h(n) = 1 - f(n). \quad (6)$$

e represents the equilibrium number of pucks per cluster for a system of 25,000 pucks.



(A) (B)

Figure 3.2: We create in (A) five stable clusters of size 5000 pucks using the functions in (B)

In this case, the variance is lower, as is expected.

These simulations indicate that the variation in the size of the clusters may be controlled by determining the behavior of the robots at the microscopic level. This is encouraging because it seems to indicate that construction may be able to occur via teams of mobile robots in a way that is both robust and reliable. Certainly it would not be reasonable to build a column or a wall that has a great variation in its size. This study indicates at least one way in which the size of the wall or column can be controlled, despite the fact that the robots need only be of minimal sensory and processing capability. This is an important first step in the development of methods of construction.

4. CONCLUSION

A number of interesting studies have motivated increased interest in the swarm engineering paradigm [2][7]. These studies have demonstrated that it is possible for simple agents to accomplish high-level tasks through the use of multiagent strategies, despite being limited in their own capability. This sparked the imagination, allowing us to dream once again of intelligent machines. This time, the machines would be groups of automatons, rather than highly intelligent agents. It is fitting to call this field *swarm intelligence*, as the final goal is the intelligent control of distributed systems.

In our current study, however, we are not after an intelligent system. We are, rather, in pursuit of a system that is capable of accomplishing engineering tasks. Moreover, we wish to determine the minimal conditions under which the given task may be accomplished. In this respect, our goal is *engineering*, not intelligence. Hence, it seems appropriate to coin the term *swarm engineering*, which underscores this intention.

The goal in our study is the creation of methods for controlling the sizes of clusters that are created as a result of the actions of swarms. Our rationale for this lies in the far off goal of distributed construction teams which behave in ways that allow the generation of three-dimensional structures of predetermined form. Certainly, the first step is to be able to generate strategies which may be undertaken by groups of mobile robots that result in the generation of clusters of specific and predetermined size, with a predetermined margin for error. This vital step is the overall goal of this study, which has determined how to reduce the variability of the size of clusters, independently of the size of the swarm.

5. REFERENCES

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