Variance in Converging Puck Cluster Sizes

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ABSTRACT

We investigate the design of control algorithms for puck clustering simulations. Of particular interest is the control of the variance in puck cluster sizes, particularly when multiple clusters are being created. We present a theoretical framework under which behaviors may be designed which serve to control puck cluster sizes. This theoretical framework is applied to non-embodied clustering systems [5], and succeeds in controlling the variance of the cluster sizes. Potential applications of such a system are discussed in Section 6. **keywords**: swarm engineering, puck clustering

1. INTRODUCTION

Puck clustering is a field of inquiry which is a subset of a much larger field of inquiry known as swarm engineering or swarm intelligence. Swarm engineering is the science of using simple agents to accomplish complex tasks. The premise is that the capabilities of the swarm exceed the individual capabilities, allowing a great number of tasks to be efficiently carried out which would have been either impossible or excessively time consuming for an individual agent of even moderate intelligence.

To date, very little work has been done on examining the underlying mathematical framework of clustering, although a great deal of work has been done on examining the mechanisms behind both biological clustering and robotic clustering [1][2][3][4][7][8][9]. For the most part, these studies examine the microscopic behaviors either employed by natural systems or designed by human designers to carry out the clustering task. While these provide tantalizing examples of the successful ways in which swarm engineering may be utilized, they do not provide a theoretical framework under which the behavior of the system may be predicted and controlled simply from correctly designing the behavior of the agents according to some predetermined methodology.

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This paper extends previous results which have been applied to puck clustering [5][6] which indicate that the development of the system may be correctly predicted by utilizing theoretical results to derive the design of the behavior of the agents in the system. Those results, which were largely theoretical in nature, demonstrated the general conditions under which clustering could occur, how to discriminate between methods of differing efficiency, and how to create multiple clusters of predetermined size ratios. This paper examines the question of controlling the variance when creating multiple clusters. We are interested in understanding the theoretical design of the behaviors of individual agents that will create the desired group behavior.

The rest of the paper is organized in the following way. Section 2 describes the computational experiments (both embodied and non-embodied) which yield multiple clusters of rather large variance. Section 3 describes the theoretical framework for generating multiple clusters with minimal variance. Section 4 describes the experimental results obtained when this method is applied to the multiple cluster systems. Section 5 examines the application of the theory to embodied simulations of robots. Finally, Section 6 provides some concluding remarks.

2. PUCK CLUSTERING IN NON-EMBODIED SIMULATIONS

In this Section, we explore the variance of puck clustering systems using a *non-embodied simulation* [1]. A non-embodied clustering simulation is one which examines the variance of clustering in the absence of embodied robotic systems. The agents are simple, having only two basic actions: choosing a random cluster, and deciding whether to pick up or drop off a puck. This allows the simulation to investigate many of the properties of puck clustering systems in advance of their implementation on embodied models.

Of course, the simplest way in which to generate a single cluster is to create and somehow mark a specific part of the work area in which the rules are to drop off pucks which have been picked up elsewhere. However, this presupposes that there is an area of interest which may be marked in some way, and that the robots have some way of determining where they are. Our approach to this same problem is minimalist. We wish to generate rules under which clustering can occur, despite the possibility that the robots might have no way of determining where they are.

2.1 Generating a Single Cluster

Initial simulations are centered around the generation of single clusters from two or more smaller clusters ranging in size from one to many pucks. In these simulations, each cluster is defined by its number of pucks. Each puck in the cluster is considered to be equally available to any agent. Each agent can hold at most one puck, and has two basic actions: choosing a cluster and deciding to pick up or drop off a puck it may be holding. The behaviors are determined by the agent's internal controller.

In these simulations, we denote the probability of picking up a puck from a cluster of size N by

$$p_u = f(N) \tag{1}$$

where f is defined by the behavioral design of the agent. We denote the probability of dropping a puck in the cluster by

$$p_d = 1 - f(N)\ . \tag{2}$$

This limits the behavior of the system by requiring that the agent behave as though it must perform a binary decision each time it encounters a cluster. This decision determines whether or not the agent considers the cluster large. If the agent considers the cluster large, then it will not pick up pucks, but rather will only drop off pucks. On the other hand, if it considers the cluster small, it will only pick up pucks. We use probabilities to simulate the likelihood that an agent in the real world will make its determination based on factors including its angle of approach of the cluster, which would allow clusters to be classified differently depending on the angle of approach.

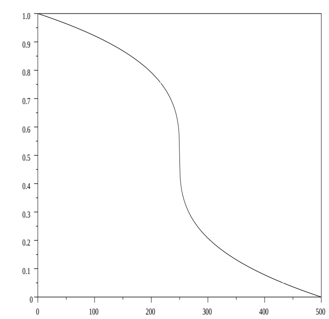
It has been shown elsewhere that the minimal condition required for a single cluster to form is that the ratio of the probabilities is a decreasing function of the number of pucks in the cluster. That is,

$$\frac{\partial}{\partial N}\left(\frac{p_u}{p_d}\right) < 0 \tag{3}$$

for all N. This in turn implies that for these probabilities,

$$\frac{\partial f}{\partial N} < 0 \tag{4}$$

An example of such a simulation is given in Figure 2.1, in which several clusters of the same initial size evolve under the action of many agents. One cluster clearly grows to dominate, and eventually absorbs all the pucks initially found in other clusters.



(B)

Figure 2.1: A non-embodied simulation during which a single cluster is generated from five initial clusters (A). The function f is depicted in (B).

Similar results can be generated using different forms of function f that are decreasing.

2.2 Generating Multiple Clusters of the Same Size

Generating single clusters is an interesting task in and of itself. However, we are interested in generating an understanding of the basic requirements for methods of clustering which may eventually be used to build predesigned structures. It has been shown elsewhere [5][6] that equal sized clusters may be obtained if

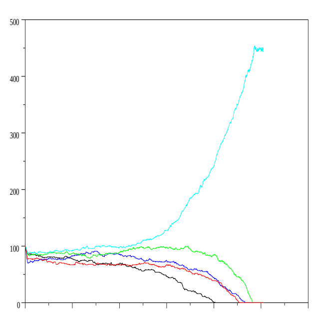
$$\frac{\partial}{\partial N}\left(\frac{p_u}{p_d}\right) \geq 0. \tag{5}$$

This condition may be obtained from the basic behaviors if the impact of the classification of the behavior is reversed. Such a reversal requires that

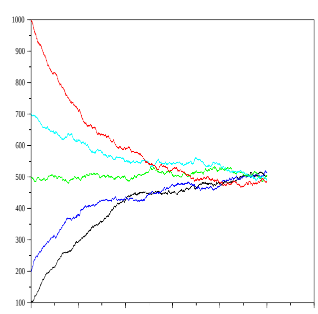
$$\frac{\partial f}{\partial N} \geq 0. \tag{6}$$

In this case, any individual clusters initially in existence will share pucks so that the clusters become equal sized.

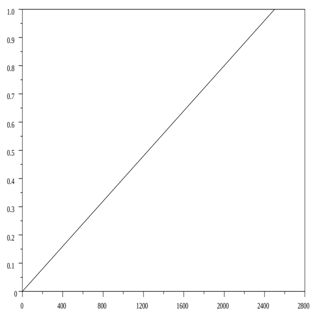
Figures 2.2 and 2.3 depict the generation of equal-sized clusters in our non-embodied model. In generating multiple equal sized clusters, pucks must be moved from the larger clusters to the smaller clusters. With the probabilistic simulation, the function f is increasing. Function h, the probability of dropping a puck, is a decreasing function equal to $1 - f$. Several clusters are initialized with different numbers of pucks.



(A)

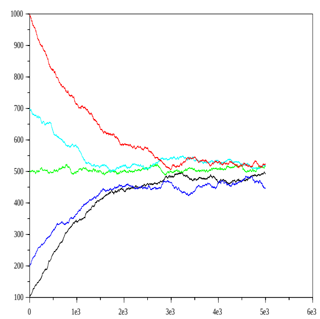


(A)

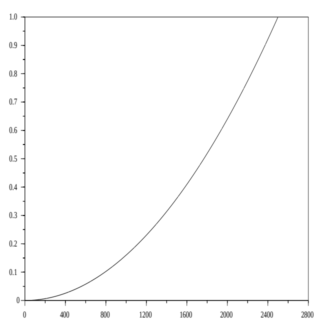


(B)

Figure 2.2: A non-embodied puck clustering simulation in which several clusters evolve to the same size.



(A)



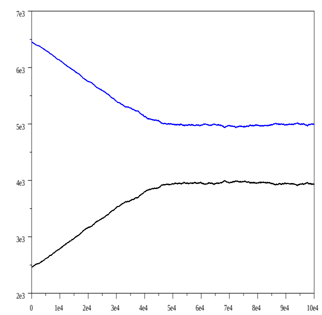
(B)

Figure 2.3: A second non-embodied puck clustering simulation in which several clusters evolve to the same size.

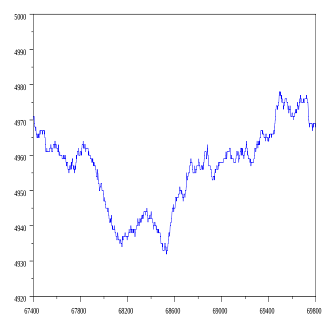
In both of these cases, the clusters converge to a single size. The rate of convergence depends on the form of f, though the final size is predetermined. In both cases, a rather large variance is observed. The variance of the different cases are different from one another. This means that, although the equilibrium sizes of the clusters are identical in either system, their dynamics are not equivalent. These dynamics depend on the detailed form of f. In Figure Thus, it stands to reason that controlling f amounts to controlling the equilibrium dynamics.

2.3 Generating Multiple Clusters of Different Sizes

We have thus far examined the development of single clusters of pucks and multiple identically sized clusters. These represent the simplest cases of clustering using any given method. However, in order to accomplish the desired task of construction, it will become necessary for us to be able to construct structures of significantly greater complexity. The first step in this is the construction of clusters of differing sizes. It has been shown [5][6] that in order to construct clusters of different sizes, the function f must have two regions, one of which is increasing, and the other of which is decreasing. This, of course, drives the creation of clusters that have a predetermined differing size. The situation is depicted in Figure 2.4 in which the clusters evolve to specific predetermined sizes (A) under the influence of the bimodal function f.



(A)



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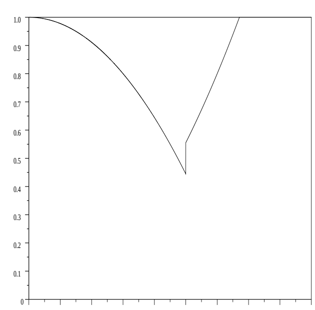


(C)

Figure 2.4: This figure depicts the evolution of a system of two clusters (A) in which the clusters evolve to two different clusters of differing sizes. The variance of the top cluster is depicted, at equilibrium, in (B). The function f which pro-

duces this behavior is depicted in (C).

This behavior is the result of the balance between the tendency to remove pucks from one cluster and the tendency to add pucks to another cluster. In the decreasing region, the tendency is to add pucks. In the increasing region, it is to drop off pucks. However, when the values of the ratio $\frac{p_u}{p_d} = \frac{f(N)}{1-f(N)}$ switch from one cluster having the larger ratio to the other having the larger ratio, the overall behavior switches from a net flow in one direction to one in the reverse direction. The equilibrium point requires that

$$f(N_1) = f(N_2) \quad (7)$$

which is the minimum necessary and sufficient condition for the equilibrium point to exist. However, in order for the two clusters to be built, the initial conditions must be such that the system evolves to these stable points.

As in the case with the clusters of identical size, the system with differing cluster sizes exhibits a significant variance. This, of course, is not desirable, as a true construction task, which is our eventual goal, will require the construction of structures of specific sizes.

3. SUPPRESSION OF VARIANCE

3.1 General Analogy

We begin our investigation of the origin of the variance seen when generating multiple clusters by considering a simple system of two clusters and dynamics which produce identical sized clusters. Without proof, we assert that our results may be generalized to systems of more than two clusters. Let n_i equal the number of pucks in cluster i. Let n_{ie} equal the number of pucks of each cluster at equilibrium, which is the average size of the system of clusters when generating equal sized clusters. Let r_t equal the total number of robots present, and r_c be the number of robots carrying pucks. Once again, f represents the probability of a robot removing a puck from a cluster as a function of the cluster size, and h is the probability of adding a puck to a cluster. The rates of change of the number of pucks in the clusters are:

$$\frac{dn_i}{dt} = -(r_t - r_c)f(n_i) + (r_c)h(n_i) \quad (8)$$

We may approximate the above equation using Taylor polynomials. This yields

$$\frac{dn_i}{dt} = -(r_t - r_c)[f(n_{ie}) + (n_i - n_{1e})f'(n_{ie})] + (\mathrm{r}_c)[h(n_{ie}) + (n_i - n_e)h'(n_{ie})] \quad (9)$$

Rearranging we obtain

$$\frac{dn_i}{dt} = [-(r_t - r_c)((n_i - n_{ie})f'(n_{ie})) + (r_c)((n_i - n_{ie})h'(n_{ie}))] + [-(\mathrm{r}_t - r_c)f(n_{ie}) + (r_c)h(n_{ie})] \quad (10)$$

Since n_e represents the number of pucks at equilibrium, and the rate of change at equilibrium should be 0, the equation can then be simplified to

$$\frac{dn_i}{dt} = (n_i - n_{ie})[-(r_t - r_c)f'(n_{ie}) + (r_c)h'(n_{ie})] \quad (11)$$

The last equations is a linear equation with restoring constant equal to $-(r_t - r_c)f'(n_{ie}) + (r_c)h'(n_{ie})$. Therefore, to increase the restoring constant it is necessary to increase the magnitude of $-(r_t - r_c)f'(n_{ie}) + (r_c)h'(n_{ie})$. This can be done by increasing the first derivatives of f and h at the predetermined equilibrium point.

3.2 Number of Puck Carrying Robots

We see that the number of robots carrying pucks, r_c, is also an element of the restoring constant in the previous analogy $-(r_t - r_c)f'(n_e) + (r_c)h'(n_e)$. However, in trials of the non-embodied simulation during which only the number of robots are altered, the variance seems unchanged.

Consider a system with P pucks in C clusters. The number of pucks held by robots at any moment is equal to the difference of the total number of pucks and the total number of pucks in the clusters at that moment:

$$r_c(t) = P - \sum_{k=1}^{C} n_k(t) \quad (12)$$

From this equation, we see that:

$$\frac{dr_c}{dt} = -(\frac{dn_1}{dt} + \frac{dn_2}{dt} + ... + \frac{dn_C}{dt}) \quad (13)$$

This shows that when the system is in equilibrium the rate of change of the number of pucks held by robots is 0. Since the rate of change of puck holding robots is 0 when in equilibrium, these robots will have no effect on the system's deviation from equilibrium. The variance is given by

$$v = \sqrt{\frac{\sum_{k=1}^{C}(n_e - n_k)^2}{C}} \ . \quad (14)$$

In variance v, the only numbers involved are clusters sizes and the number of clusters. Since the equilibrium point, number of pucks in each cluster, and number of clusters, are all independent of the number of robots, our variance would not seem to depend at all on the number of robots in the system. Moreover, since n_e simply depends on the design of the probabilities p_u and p_d, the variance must be a property of the system as defined by p_u and p_d, rather than a property of the system including the robots.

4. APPLICATION OF THEORY TO NON-EMBODIED SIMULATION

In the previous Section, we determined the condition required to restrict the variance of the sizes of the puck clusters under the action of the robots. We now apply this condition to a non-embodied clustering system. Each simulation consists of construction of identical-sized clusters. In these simulations, we calculate the variance as given in (18) for a period of time after the system has converged on equilibrium.

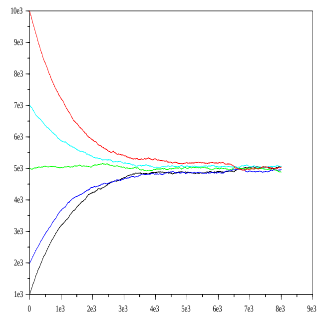
First, let

$$f_1(n) = \frac{n}{n_t} \quad (15)$$

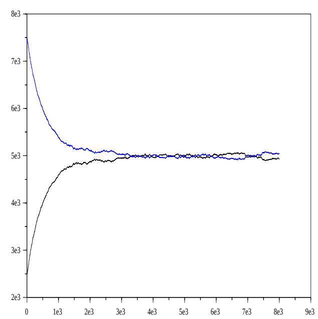
with n equal to the number of pucks in the cluster encountered by the robot, and n_t equal to the total number of pucks in all clusters. Also, let

$$h_1(n) = 1 - f_1(n) \ . \quad (16)$$

This, of course, creates several clusters of the same size. The system's evolution is as depicted in Figure 4.1 under two sets of initial conditions.



(A)



(B)

Figure 4.1: These figures depict the evolution of a system of clusters of pucks under the influence of agents characterized by the functions f and h given in (19) and (20). In (A), the system is initialized with five differently sized clusters, while in (B), the system is initialized with two differently sized clusters. Both simulations produce clusters of identical size.

We find and record the average variance of each simulation over the last 100,000 iterations. This is repeated 100 times, and the average is reported.

The average of the recorded variance for 100 simulations in which five identically sized clusters are generated from five clusters of initial sizes 1000, 2000, 5000, 7000, and 10000 pucks, using 100 robots, and subject to the f_1 and h_1 functions in equations (19) and (20) is 23.992666. That for 100 simulations in which two clusters are generated from initial sizes of 2500 and 7500 pucks is 18.938815.

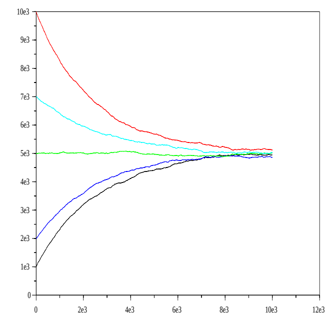
Let us now suppose that

$$f_2(n) = \frac{f_1(n)}{2} = \frac{n}{2n_t} \tag{17}$$

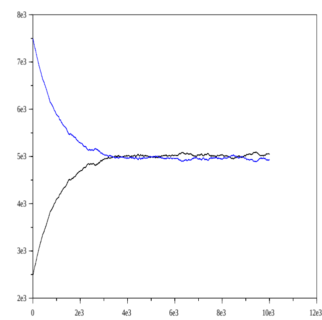
and

$$h_2(n) = \frac{h_1(n)}{2} = \frac{1}{2} - f_2(n) \tag{18}$$

The effect of halving both functions f and h is that the probabilities of picking up and dropping off pucks at clusters is halved. This would seem to imply that there will be less interaction between robots and pucks, which in turn implies that the rate of convergence will diminish. This fits nicely in our previous discussion which indicates that the variance should increase, as the restoring force is effectively halved at equilibrium.



(A)



(B)

Figure 4.2: These figures depict the evolution of a system of clusters of pucks under the influence of agents characterized by the functions f and h given in (21) and (22). In (A), the system is initialized with five differently sized clusters, while in (B), the system is initialized with two differently sized clusters.

In this case, the variance in producing five equal sized clusters from original clusters of sizes given above is 24.590322. The same quantity is 21.103539 when generating two identically sized clusters from initial conditions given above. In both cases, the variance is higher than the variance calculated when our agents used f_1 and h_1, as expected, though that for five clusters has a rather small change from the variance observed for the earlier simulations.

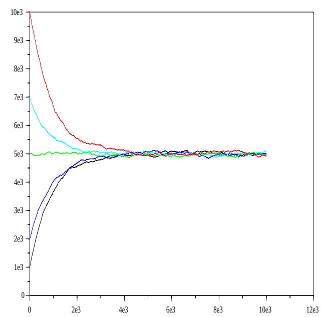
Lower variances should be able to be obtained by utilizing functions with higher first derivatives at the equilibrium point, as indicated by the considerations of Section 3. We may examine this using the functions

$$f_3(n) = \begin{cases} \frac{n - low}{2000} + \frac{1}{2} & \text{if } low < n < high \\ \frac{n}{4(low)} & \text{if } n \leq low \\ \frac{n - high}{4(n_t - high)} & \text{if } n \geq high \end{cases} \tag{19}$$

and

$$h_3(n) = 1 - f_3(n). \tag{20}$$

where n represents the cluster size, e represents the equilibrium number of pucks per cluster, $high$ is set equal to $e + 500$ and low is set equal to $e - 500$. These functions, depicted in Figure 4.4, cannot be strictly linear, and consist of varying increasing regions. The reason for using two functions is that the high slope area must occur at differing equilibrium points in either case, and thus one is not applicable to the other. That is, the equilibrium points for each of the two different simulations are not identical, and so the two simulations require differing functions.



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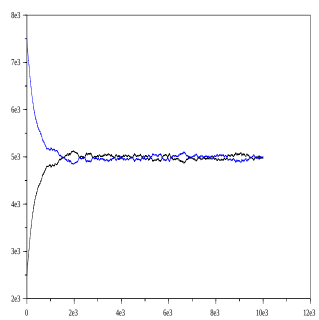
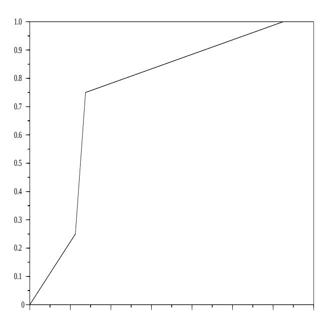
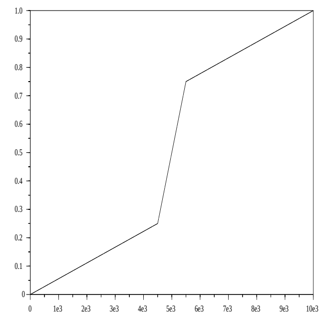


(B)

Figure 4.3: We utilize the function in (A) to create five stable clusters of size 5000 pucks while we utilize the function in (B) to generate two stable clusters of size 10000 pucks.



(A)



(B)

Figure 4.4: (A) Five clusters of equal size and reduced variance are created using agents whose behavior is commensurate with Figure 4.3 (A). (B) Two clusters of equal size and reduced variance are created using agents whose behavior is commensurate with Figure 4.3 (B).

With the new f function, we find that the average variance for generating five clusters is 14.298499, and the average variance for generating two clusters is 14.583470. In both cases, the variance is lower, as is expected.

These simulations indicate that the variation in the size of the clusters may be controlled by simply determining the behavior of the robots at the microscopic level. Though we do not provide data here, similar results have been obtained on dissimilar sized clusters. This is encouraging because it would seem to indicate that construction may be able to occur via teams of mobile robots in a way that is both robust and reliable. Certainly it would not be reasonable to build a column or a wall that has a great variation in its size. This study indicates at least one way in which the size of the wall or column can be controlled, despite the fact that the robots need only be of minimal sensory and processing capability. This is an important first step in the development of methods of construction.

5. EMBODIED SIMULATION

Theoretical studies are extremely interesting and important, as they can indicate ways of solving problems that defy our intuition and imagination. Indeed, it is not clear that the development of the methods we have examined here for the development of construction techniques would have been possible to generate simply through one's imagination. However, it is even more important that these theoretical results can be clearly demonstrated in a real-world environment. While these studies are currently in progress, a next best approach is the use of embodied simulations. Such simulations have many properties of the real world. The sensing capability of the robots to be used may be simulated, as well as the movements and sizes of the robots. In very advanced cases, even the detailed behavior of robot parts including grippers, actuators, etc. may be accurately simulated.

We explore a puck clustering system using a two dimensional embodied simulation. We apply our formalism to the robots in this simulation, in order to examine the applicability of the theory to a more realistic environment. Robots, pucks, and the arena have sizes in this simulation. Each robot can hold exactly one puck, and each robot has three basic actions: deciding to drop off a puck, deciding to pick up a puck, and moving around in the arena.

In this simulation, robots and pucks are circular in shape; the robots have a larger radius than the pucks. Robots are able to see pucks within a given distance, and within a viewing cone. This provides only information on how many pucks a robot can see, however. When the simulation is initialized, robots and pucks are placed randomly in a walled arena, as in Figure 5.2(A).

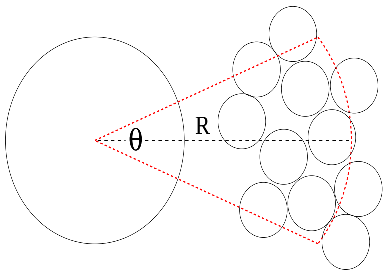
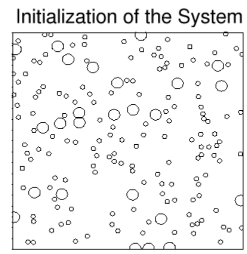


Figure 5.1: This is a typical screen-shot of the initial state of a puck clustering experiment. The larger circles represent robots, while the smaller circles represent pucks. Θis the viewing angle of the robot, and R is the viewing distance.

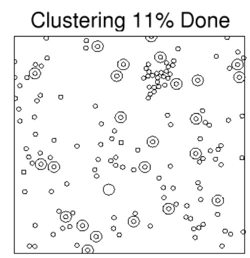
During the course of this simulation, robots move in straight lines until encountering an object. If the robot encounters another robot or an arena wall, it executes a random turn and continues to move in straight lines. If the object is a puck, the robot will decide whether to drop pick it up (if it's not currently holding a puck) or drop off the puck it's holding (if indeed it is holding a puck) based on what it perceives to be the local density. Robots are not able to see the entire cluster, but they can estimate the cluster size based on the number of pucks in their visual field. If neither of the preconditions exist under the appropriate post conditions, the robot will simply turn randomly and move away in a straight line as in the case of encountering a wall or other robot.

As in the non-embodied simulation, the probability of a robot picking up a puck is p_u and the probability of a robot dropping off a puck is $1 - p_u$. However, the value of p_u is determined by the size of the cluster and the probability that the number of pucks in the visual field exceeds a specific minimum number. If the number of pucks in the visual field exceeds the specific number, the robot considers itself to be in a high density area, and will drop off a puck, if it is holding one. However, if the number of pucks in the visual field is below a given threshold, the robot will not drop off pucks, but rather pick up a puck if it is not holding one. As the estimation of the size is highly dependent on the angle with which the robot encounters the cluster, there is a decreasing probability that the robots will incorrectly classify the clusters. This is a precondition, as determined above, for a single cluster to be built.

When this behavior is applied to the robots within an embodied simulation, it is expected that it will lead to a single cluster, as indicated by the theoretical analysis. A typical simulation in which these conditions are utilized is illustrated in Figure 5.2.



(A)



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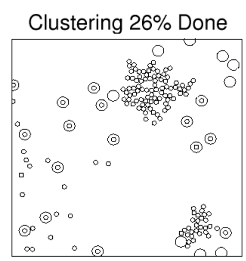

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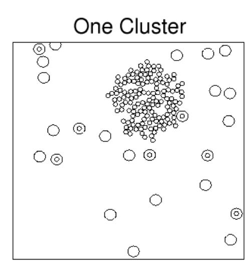

(D)

Figure 5.2: This is a typical clustering simulation in which the robot utilizes an estimated number of pucks to generate a decision about whether or not it will alter the cluster. This microstrategy leads to an overall clustering effect, as indicated by the theoretical consideration of previous sections and by considerations presented in [1].

In this Figure, the state of the system is increasingly orderly, with an emergent cluster of pucks absorbing all the pucks in the arena, except those carried by robots. This is extremely encouraging, as it indicates that the theoretical results may be applied to multirobot simulations, and will generate the expected results. Moreover, this type of simulation may be used to quickly verify behaviors that may then be transferred to real robotic platforms.

6. CONCLUSION

A number of interesting studies have motivated increased interest in the swarm engineering paradigm [2][7][8]. These studies have demonstrated that it is possible for simple agents to accomplish high-level tasks through the use of multiagent strategies, despite being limited in their own capability. This sparked the imagination, allowing us to dream once again of intelligent machines. This time, the machines would be groups of automatons, rather than highly complex intelligent agents. It is, of course, fitting to call this field *swarm intelligence*, as the final goal was the intelligent control of distributed systems. Indeed, this effect has been recognized even in daily life; the capitalist "unseen hand" is itself an emergent property of the interactions between agents, and is seen to be capable of exacting intelligent decisions on a market economy.

In our current study, however, we are not after an intelligent system. We are, rather, in pursuit of a system that is capable of accomplishing engineering tasks. Moreover, we wish to determine the minimal conditions under which the given task, in this case puck clustering, may be accomplished. In this respect, our goal is *engineering*, not intelligence. Hence, it seems appropriate to coin the term *swarm engineering*, which underscores this intention.

In this study, our goal is the creation of methods for controlling the size of clusters that are created as a result of the actions of swarms of agents. Our rationale for this lies in the far off goal of distributed construction teams which behave in ways that allow the generation of three-dimensional structures of predetermined form. "Clusters" are simply collections of objects, no matter the detailed structure of the clusters. For instance, Martinoli et. al. [8] have demonstrated methods for creating clusters of significantly different design than those we are utilizing. In the future, clusters may refer to structures such as columns, floors, staircases, etc., and our goal is the understanding of ways in which to build these structures. Certainly, the first step is to be able to generate strategies which may be undertaken by groups of mobile robots that result in the generation of clusters of specific and predetermined size, with a predetermined margin for error. This vital step is the overall goal of this study, which has determined how to reduce the variability of the size of clusters, independently of the size of the swarm.

The next step in this research is the investigation of the application of these techniques to embodied simulations capable of generating multiple clusters of predefined size and distribution. In our theoretical studies, it is possible to generate these conditions based on the assumption that large clusters will not be separated into smaller clusters. However, embodied simulations cannot guarantee this constraint, which makes the goal significantly more elusive. More study is required in order to generate ways of automatically controlling the generation of multiple clusters.

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